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QUESITI 8 / 1 / 2014

4 $f : \mathbb{R}^7 \to \mathbb{R}^4$ $\qquad \underline{7e_1 - e_2 + e_4} \notin \operatorname{Im} f$

$\dim \operatorname{Ker} f = ?$

$0 \leq \dim \operatorname{Im} f \leq 3$

$4 \leq \dim \operatorname{Ker} f = 7 - \dim \operatorname{Im} f \leq 7$

QUESITI 27/1/2019 (VER. I)

ES. 4 $\quad p(z) = 2z^3 - (1+3i)z^2 + 2iz + (1-i) \qquad p(i)=0$

$\Rightarrow$ divido $p(t)$ per $(x-i)$

2	$-(1+3i)$	$2i$	$(1-i)$	$x-i$
2	$-2i$			
	$-(1+i)$	$2i$	$(1-i)$	$2x^2-(1+i)x+(1+i)$
	$-(1+i)$	$(-1+i)$		
		$(1+i)$	$(1-i)$	
		$(1+i)$	$(1-i)$	

$q(x) = 2x^2 - (1+i)x + (1+i)$

$q(i) = 0 \qquad q(x) = 2(x-i)\left(x-\frac{1}{2}(1-i)\right)$

dividendo ancora

$\Rightarrow p(x) = 2(x-i)^2\left(x-\frac{1}{2}(1-i)\right)$

Es. 5

$f : \mathbb{C}^4 \longrightarrow \mathbb{C}^7$

$f(e_1 + i e_3) = f(i e_1 + e_2 + (1-i) e_4)$

$= (5-i) e_4 + e_7$

perchè due vettori hanno la stessa immagine

$\dim \operatorname{Ker} f \geq 1$

almeno un vettore $\neq 0$ sta nell'immagine

$\dim \operatorname{Im} \geq 1$

$\Downarrow$

$\dim \operatorname{Ker} \leq 3 = 4 - 1$

$\dim \operatorname{Ker} + \dim \operatorname{Im} = 4 \qquad - \dim \operatorname{Ker} + 4 \geq 1$

$1 \leq \dim \operatorname{Ker} f \leq 3$

Es. 6

$$\begin{pmatrix} 1 & 1 & -1 \\ 2 & 4 & 1 \\ -1 & 1 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 7 \\ 10 \\ -11 \end{pmatrix}$$

$$\det = 15 - 9 - 6 = 0$$

$$\underline{\mathrm{II}} - 3\,\underline{\mathrm{I}} = \underline{\mathrm{III}} \qquad (\text{righe})$$

$$a \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} + b \begin{pmatrix} 1 \\ 4 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ 4 \end{pmatrix}$$

$$\det \begin{pmatrix} 1 & 1 \\ 2 & 4 \end{pmatrix} = 2$$

$$a = \frac{1}{2} \begin{vmatrix} -1 & 1 \\ 1 & 4 \end{vmatrix} = -\frac{5}{2} \qquad b = \frac{1}{2} \begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix} = \frac{3}{2}$$

$$\ker = \left\langle \left(-\frac{5}{2}, \frac{3}{2}, -1 \right) \right\rangle$$

$$\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} x + \begin{pmatrix} 1 \\ 4 \\ 1 \end{pmatrix} y = \begin{pmatrix} 7 \\ 10 \\ -4 \end{pmatrix} =$$

$$x = \frac{1}{2} \begin{vmatrix} 7 & 1 \\ 10 & 4 \end{vmatrix} = 7 \qquad y = \frac{1}{2} \cdot \begin{vmatrix} 1 & 7 \\ 2 & 10 \end{vmatrix} = -2$$

vettore del nucleo

$$\begin{cases} x = 7 - 5t \\ y = -2 + 3t \\ z = -2t \end{cases}$$

sol particolare

ES. 7 $\quad X = \text{Span}\left\langle \begin{pmatrix} 1 \\ 0 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 3 \\ -1 \end{pmatrix} \right\rangle = \langle w_1, w_2 \rangle$

$$\mathbb{R}^4 = X \oplus Y$$

$$Y = \text{Span}\left\langle \begin{pmatrix} 2 \\ -1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 4 \\ -1 \\ -1 \\ 0 \end{pmatrix} \right\rangle$$

$$v = \begin{pmatrix} 9 \\ -4 \\ -2 \\ 4 \end{pmatrix}$$

trovare la proiezione di v su X.

Equazioni per Y:

$$\begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} \qquad x + y + az + bw = 0$$

$$\begin{pmatrix} 2 & -1 & 0 & 1 \\ 4 & -1 & -1 & 0 \\ x & y & z & w \end{pmatrix}$$

$$x \cdot 1 + y \cdot 2 + z \cdot 2 = 0$$

$$x \cdot 1 + y \cdot 4 + w \cdot 2 = 0$$

$$f_1(x,y,z,w) = x + 2y + 2z$$

$$f_2(x,y,z,w) = x + 4y + 2w$$

$$f_1(v) = f_1(9,-4,-2,4) = -3$$

$$f_2(v) = 1$$

$$f_1(w_1) = 5 \qquad f_2(w_1) = 3$$

$$f_1(w_2) = 8 \qquad f_2(w_2) = 2$$

$$v_x = \pi_X(v) = \alpha w_1 + \beta w_2$$

t.c. $f_1(v_x) = f_1(v)$

$f_2(v_x) = f_2(v)$

$$-3 = 5\alpha + 8\beta \qquad \det = -14$$

$$+1 = 3\alpha + 2\beta$$

$$\alpha = -\frac{1}{14} \cdot \begin{vmatrix} -3 & 8 \\ 1 & 2 \end{vmatrix} = -\frac{1}{14}(-14) = 1$$

$$\beta = -\frac{1}{14} \cdot \begin{vmatrix} 5 & -3 \\ 3 & 1 \end{vmatrix} = -\frac{1}{14} \cdot 14 = -1$$

$$V_x = w_1 - w_2 =$$

$$= \begin{pmatrix} 1 \\ 0 \\ 2 \\ 1 \end{pmatrix} + \begin{pmatrix} 0 \\ -1 \\ -3 \\ +1 \end{pmatrix} = e_1 - e_2 - e_3 + 2e_4$$