# Alg Lin 16/12/14

15/2/14 - Variante 0 - Quesito 3

$X, Y \subset \mathbb{C}^{12}$, $\dim X = 8$, $\dim Y = 9$ ; $\dim(X \cap Y) = ?$

$$\underset{8}{\dim X} + \underset{9}{\dim Y} = \underset{\leq 12}{\dim(X+Y)} + \dim(X \cap Y).$$

$\Rightarrow \dim(X \cap Y) \geq 8 + 9 - 12 = 5$ ; inoltre $X \cap Y \subset X$

$\Rightarrow \dim(X \cap Y) \leq 8 \Rightarrow$ tra 5 e 8 compresi

(Tutti i casi tra 5 e 8 si realizzano:

$X = \mathbb{C}^8 \times \{(0,0,0,0)\}$

$Y = \{(0,0,0)\} \times \mathbb{C}^9$

analog. gli altri _)

$X \cap Y = \{(0,0,0)\} \times \mathbb{C}^5 \times \{(0,0,0,0)\}$

$\Longrightarrow \dim = 5$

$\boxed{41}$ Q3 (pag. 33) $X = \{x \in \mathbb{R}^3 : x_1 + x_2 + x_3 = 0\}$

$f : X \to X \quad f(x) = \begin{pmatrix} 2x_1 - x_2 \\ x_3 + x_2 \\ x_2 - x_1 \end{pmatrix}$

$\mathcal{B} = \left( \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \right)$ : trovare $[f]^{\mathcal{B}}_{\mathcal{B}}$ :

$$f\begin{pmatrix}1\\1\\-2\end{pmatrix}=\begin{pmatrix}1\\-1\\0\end{pmatrix}=-1\begin{pmatrix}1\\1\\-2\end{pmatrix}+2\begin{pmatrix}1\\0\\-1\end{pmatrix}$$

$$f\begin{pmatrix}1\\0\\-1\end{pmatrix}=\begin{pmatrix}2\\-1\\-1\end{pmatrix}=-1\begin{pmatrix}1\\1\\-2\end{pmatrix}+3\begin{pmatrix}1\\0\\-1\end{pmatrix}$$

$$\Longrightarrow\begin{pmatrix}-1 & -1\\2 & 3\end{pmatrix}$$

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Fogl. 6/12/14 es. 8

(a) $E: (1-4i)z_1+(2-i)z_2+(3i-2)z_3=4-5i \subset \mathbb{C}^3$

$\dim = 3-1=2$

$$\begin{pmatrix} \frac{4-5i}{1-4i} \\ 0 \\ 0 \end{pmatrix} \operatorname{Span}\left( \begin{pmatrix} i-2 \\ 1-4i \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 3i-2 \\ i-2 \end{pmatrix} \right)$$

$$\frac{4-5i}{1-4i} = \frac{1}{17}(4-5i)(1+4i) = \frac{1}{17}(24+7i)$$

(c)
$$\begin{cases} (1-i)z_1 + 2iz_2 + (4-i)z_3 = i \\ 2iz_1 + (1+2i)z_2 + (1+i)z_3 = -2 \end{cases}$$

$$\dim = 3-2 = 1$$

$$\frac{1}{\det\begin{pmatrix}1-i & 2i\\ 2i & 1+2i\end{pmatrix}}\begin{pmatrix}\det\begin{pmatrix}i & 2i\\ -2 & 1+i\end{pmatrix}\\ \det\begin{pmatrix}1-i & i\\ 2i & -2\end{pmatrix}\\ 0\end{pmatrix}+\mathrm{Span}\begin{pmatrix}\det\begin{pmatrix}2i & 4-i\\ 1+2i & 1+i\end{pmatrix}\\ -\det\begin{pmatrix}1-i & 4-i\\ 2i & 1+i\end{pmatrix}\\ \det\begin{pmatrix}1-i & 2i\\ 2i & 1+2i\end{pmatrix}\end{pmatrix}$$

[calcoli]

(d) $\begin{cases}1+i & 3i & 3-2i & 5 & =2i\\ 2+i & 2i & 1-2i & 3i & =1\end{cases}$

$\dim = 4-2=2$

$$\frac{1}{\det\begin{pmatrix}1+i & 3i\\ 2+i & 2i\end{pmatrix}} \begin{pmatrix} \det\begin{pmatrix}2i & 3i\\ 1 & 2i\end{pmatrix} \\ \det\begin{pmatrix}1+i & 2i\\ 2+i & 1\end{pmatrix} \\ 0 \\ 0\end{pmatrix} + \operatorname{Span}\left( \begin{pmatrix} \det\begin{pmatrix}3i&5\\2i&3i\end{pmatrix} \\ -\det\begin{pmatrix}1+i&5\\2+i&3i\end{pmatrix} \\ 0 \\ \det\begin{pmatrix}1+i&3i\\2+i&2i\end{pmatrix}\end{pmatrix} \begin{pmatrix}0\\ \det\begin{pmatrix}3-2i&5\\1-2i&3i\end{pmatrix} \\ -\det\begin{pmatrix}3i&5\\2i&3i\end{pmatrix} \\ \det\begin{pmatrix}3i&3-2i\\2i&1-2i\end{pmatrix}\end{pmatrix}\right)$$

(8f) $E = \begin{pmatrix}1-i\\3i-2\\-4i\\-4\end{pmatrix} + \operatorname{Span}\begin{pmatrix}6+i\\1+i\\2-i\\5+i\end{pmatrix}$ $\quad$ $\dim = 1$.

$$\begin{cases}(i-2)z_1+(6+i)z_3 = 5-27i\\ (i-2)z_2+(1+i)z_3 = \dots\\ (5+i)z_3+(i-2)z_4 = \dots\end{cases}$$

$$(i-2)(1-i)+(6+i)(-4i)=$$

$$=-1-3i-24i+6=5-27i$$

(h) $\begin{pmatrix}-1\\ i\\ 1+i\\ 1\end{pmatrix}+\mathrm{Span}\left(\begin{pmatrix}i\\ 3-i\\ 4-i\\ 2+i\end{pmatrix}\begin{pmatrix}1+i\\ 2\\ 3i\\ i-3\end{pmatrix}\right)$ $\dim=2$

$\Longrightarrow 4-2=2$ equaz.

$$\begin{cases}\det\begin{pmatrix}3-i & 2\\ 4-i & 3i\end{pmatrix}\cdot z_1-\det\begin{pmatrix}i & 1+i\\ 4-i & 3i\end{pmatrix}z_2+\det\begin{pmatrix}i & 1+i\\ 3-i & 2\end{pmatrix}\cdot z_3+0\cdot z_4=\dots\\ 0\cdot z_1+\det\begin{pmatrix}4-i & 3i\\ 2+i & i-3\end{pmatrix}\cdot z_2-\det\begin{pmatrix}3-i & 2\\ 2+i & i-3\end{pmatrix}z_3+\det\begin{pmatrix}3-i & 2\\ 4-i & 3i\end{pmatrix}z_4=\dots\end{cases}$$

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Quesiti compito 8/1/14 :

1. Dati 9 generatori di $\{p(t) \in \mathbb{C}_{\leq 6}[t] : p(1) = p'(1) = 0\}$ quanti scartarne per base?

$\dim = 7 - 2 = 5$ ; Risp: $9 - 5 = 4$ _

2. $\mathcal{B} = (v_1, v_2)$ $\quad v_1 = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$ $\quad \left[\begin{pmatrix} 5 \\ -6 \end{pmatrix}\right]_{\mathcal{B}} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$ ; $v_2 = ?$

$$\begin{pmatrix} 5 \\ -6 \end{pmatrix} = 4 \cdot v_1 + 3 v_2 = 4 \begin{pmatrix} 2 \\ -3 \end{pmatrix} + 3 \cdot v_2$$

$$\Rightarrow \quad v_2 = \begin{pmatrix} -1 \\ 2 \end{pmatrix}.$$

3. Trovare $z \in \mathbb{C}$ t.c. $\det \begin{pmatrix} 1 & -z & i \\ 0 & -1 & 1 \\ -i & 0 & i \end{pmatrix} = 0.$

$$z \cdot \det \begin{pmatrix} 0 & 1 \\ -i & i \end{pmatrix} - \det \begin{pmatrix} 1 & i \\ -i & i \end{pmatrix} = 0$$

$$z = \frac{i-1}{i} = (-i)(i-1) = 1+i.$$

4. Risolvere $\begin{cases} 3x + y - 2z = 1 \\ 2x - y + 3z = 10 \\ -x + 4y + 5z = -9. \end{cases}$

TROVARE TUTTE E SOLE LE SOLUZIONI

$$\begin{cases} y = 2x + 3z - 10 \\ 5x + z = 11 \\ \cancel{13x - 13z = 13} \end{cases} \qquad \begin{cases} x = 2 \\ z = 1 \\ y = -3 \end{cases}$$

$$x - z = 1$$

6. $A = \begin{pmatrix} 2 & 0 & -1 \\ 1 & -2 & 1 \\ 2 & 5 & -1 \end{pmatrix}$ : $(A^{-1})_{23}$

$$\det A = 4 - 5 - 4 - 10 = -15$$

$$(A^{-1})_{23} = \frac{(-1)}{-15} \det \begin{pmatrix} 2 & -1 \\ 1 & 1 \end{pmatrix} = \frac{3}{15} = \frac{1}{5}.$$

7. $X = \{ x \in \mathbb{R}^3 : 5x_1 + 3x_2 - 4x_3 = 0 \}$ $Y = \mathrm{Span}\begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$

$\mathbb{R}^3 = X \oplus Y$

Proiez. su $X$ di $\begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix}$.

Cerchiamo $\begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix} = \underset{X}{\underset{\cap}{x}} + \underset{Y}{\underset{\in}{y}}$

cioè $y = t \cdot \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$

Cioè vogliamo che $\begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix} - t \cdot \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$ stia in $X$,

cioè che soddisfi l'equaz. di $X$:

$$(5,3,-4)\Bigg(\underbrace{\begin{pmatrix}2\\1\\-3\end{pmatrix}-t\begin{pmatrix}2\\1\\2\end{pmatrix}}\Bigg)=0$$

trovato t, la proiez. cercata è lui

$$(10+3+12)=t\,(10+3-8) \qquad t=5$$

$$\Rightarrow \begin{pmatrix}2\\1\\-3\end{pmatrix}-5\begin{pmatrix}2\\1\\2\end{pmatrix}=\begin{pmatrix}-8\\-4\\-13\end{pmatrix}.$$

Quesiti 27/1/14 :

1. $f: V \to W$ invertibile ; calcolare $\det [f^{-1}]_{\mathcal{C}}^{\mathcal{B}}$ sapendo $\det \left( [f]_{\mathcal{B}}^{\mathcal{C}} \right)$.

Fatto : $[f^{-1}]_{\mathcal{C}}^{\mathcal{B}} = \left( [f]_{\mathcal{B}}^{\mathcal{C}} \right)^{-1}$

$$\Rightarrow \det \left( [f^{-1}]_{\mathcal{C}}^{\mathcal{B}} \right) = \frac{1}{\det \left( [f]_{\mathcal{B}}^{\mathcal{C}} \right)} .$$

2. $\mathcal{B} = \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix}$ base di $\{ 5x_1 + 6x_2 - x_3 = 0 \}$

$$\left[ \begin{pmatrix} 1 \\ 1 \\ 11 \end{pmatrix} \right]_{\mathcal{B}} = ?$$

$$\begin{pmatrix} 1 \\ 1 \\ 11 \end{pmatrix} = 3 \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} + 2 \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix} \Longrightarrow \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

3. $A = (v_1, v_2)$ $\quad B = \left( (1+i)v_1 + 3v_2, (2-i)v_1 + (1+3i)v_2 \right)$

$\det(A) = 1-i$ ; $\quad \det B = ?$

$$B = (v_1, v_2) \cdot \begin{pmatrix} 1+i & 2-i \\ 3 & 1+3i \end{pmatrix}$$

$$\Rightarrow \det B = \det(v_1, v_2) \cdot \det \begin{pmatrix} 1+i & 2-i \\ 3 & 1+3i \end{pmatrix}$$

$$= (1-i)(-2+4i-6+3i) = (1-i)(-8+7i)$$

$$= -1+15i$$