Esercitazione Alg Lin 11/11/2014

Foglio 2/11/2014

<u>Es</u>. 2

$$A = \begin{pmatrix} 2 & -1 & -2 \\ 3 & 4 & 5 \end{pmatrix}$$

$$f_{(A \cdot B)}(x) = (f_A \circ f_B)(x) \qquad B = \begin{pmatrix} 5 & -3 \\ 1 & 4 \\ 2 & -1 \end{pmatrix} \qquad x = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$$

$$A \cdot B = \begin{pmatrix} 5 & -8 \\ 23 & 2 \end{pmatrix}$$

$$f_{(A \cdot B)}(x) = \begin{pmatrix} 5 & -8 \\ 23 & 2 \end{pmatrix} \begin{pmatrix} -2 \\ 3 \end{pmatrix} = \begin{pmatrix} -34 \\ -52 \end{pmatrix}$$

$$f_B(x) = \begin{pmatrix} 5 & -3 \\ 1 & 4 \\ 2 & -1 \end{pmatrix} \begin{pmatrix} -2 \\ 3 \end{pmatrix} = \begin{pmatrix} -19 \\ 10 \\ -7 \end{pmatrix}$$

$$f_A\big(f_B(x)\big) = \begin{pmatrix} 2 & -1 & -2 \\ 3 & 4 & 5 \end{pmatrix} \begin{pmatrix} -19 \\ 10 \\ -7 \end{pmatrix} = \begin{pmatrix} -34 \\ -52 \end{pmatrix}$$

Es 3 $\quad A \in M_{m \times n}(\mathbb{R}) \qquad B \in M_{k \times h}(\mathbb{R})$

$$f(X) = A\,X\,B \qquad f : M_{n \times k}(\mathbb{R}) \longrightarrow M_{m \times h}(\mathbb{R})$$

lineare

$$AX \in M_{m \times k}(\mathbb{R})$$

$$(AX)B \in M_{m \times h}(\mathbb{R})$$

linearità $f(aX+bY) =$

$= A(aX+bY)B = aAXB + bAYB =$

$= a\, f(X) + b\, f(Y)$ ok

$\mathrm{Ker}(f) = \{ X \in M_{n \times k}(\mathbb{R}) : X(\mathrm{Im}\, B) \subset \mathrm{Ker}\, A \}$

$B = (B^1, \dots, B^h)$ $\mathrm{Im}\, B = \mathrm{Span}(B^1, \dots, B^h)$

$X(\mathrm{Im}\, B) = \mathrm{Span}(XB^1, \dots, XB^h)$

$X(\mathrm{Im}\, B) \subset \mathrm{Ker}\, A$ $\Longleftrightarrow$

$$\iff \quad A(XB^1) = 0, \quad A(XB^2) = 0, \quad \dots, \quad A(XB^q) = 0$$

$$A(XB^1, \dots, XB^q) = 0$$

$$(A(XB^1), \dots, A(XB^q)) = 0$$

$$AXB = 0$$

quindi $\quad X \in \operatorname{Ker}(f) \iff X(\operatorname{Im} B) \subset \operatorname{Ker} A$.

<u>Es</u> 4. $\quad X = \{ x \in \mathbb{R}^3 : x_1 + x_2 + x_3 = 0 \}$

$$A = \left(\qquad \right),$$

$g(x) = A \cdot x$ notiamo che $g : X \to X$

A definisce una applicazione lineare

$$\mathbb{R}^3 \to \mathbb{R}^3$$
$$\cup$$
$$X$$

A definisce $X \longrightarrow \mathbb{R}^3$ lineare

$x \in \mathbb{R}^3$ $\quad x = (x_1, x_2, x_3)$

$$Ax = x_1 \begin{pmatrix} 4 \\ -7 \\ 1 \end{pmatrix} + x_2 \begin{pmatrix} -3 \\ 1 \\ 2 \end{pmatrix} + x_3 \begin{pmatrix} 3 \\ -9 \\ 4 \end{pmatrix}$$

faccio la somma delle coordinate di Ax

$$x_1(4-7+1)+x_2(-5+1+2)+x_3(3-9+4)=$$

$$=x_1(-2)+x_2(-2)+x_3(-2)=$$

$$=-2(x_1+x_2+x_3)$$

$$Ax\in X \iff x_1+x_2+x_3=0$$

$$\Updownarrow$$

$$x\in X$$

Quindi se $x \in X$ $\quad Ax \in X$

e dunque A definisce un'appl lineare

$X \longrightarrow X$.

<u>Es 5</u> $\quad A = \begin{pmatrix} 2 & 1 \\ 3 & -4 \end{pmatrix}$

$X = \{ B \in M_{2\times 2}(\mathbb{R}) \ : \ AB = BA \}$

$B = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ $\qquad AB = BA$

$$\begin{pmatrix} 2a + c & 2b + d \\ 3a - 4c & 3b - 4d \end{pmatrix} = \begin{pmatrix} 2a + 3b & a - 4b \\ 2c + 3d & c - 4d \end{pmatrix}$$

$\rightarrow$ sistema lineare $\Rightarrow$ X è sottosp vett. di $M_{2\times 2}(\mathbb{R})$

$$\begin{cases} \cancel{2a} + c = \cancel{2a} + 3b \\ 3a - 4c = 2c + 3d \\ 2b + d = a - 4b \\ 3b - \cancel{4d} = c - \cancel{4d} \end{cases}$$

$$\begin{cases} c = 3b \\ 3a - 6c - 3d = 0 \\ 6b + d - a = 0 \\ 3b = c \end{cases} \quad (\text{eq } I^e)$$

$$\begin{cases} 3b = c \\ a - 2c - d = 0 \end{cases}$$

$2c + d - a = 0$ multiplo

$\dim \{B : AB = BA\} = 2$

Es 6

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \qquad B = \begin{pmatrix} x_1 & x_2 \\ x_3 & x_4 \end{pmatrix}$$

$$AB = BA \iff \begin{cases} \cancel{a x_1} + b x_3 = \cancel{a x_1} + c x_2 \\ a x_2 + b x_4 = b x_1 + d x_2 \\ c x_1 + d x_3 = a x_3 + c x_4 \\ c x_2 + \cancel{d x_4} = b x_3 + \cancel{d x_4} \end{cases}$$

$$\begin{cases} b x_3 = c x_2 \\ a x_2 + b x_4 = b x_1 + d x_2 \\ c x_1 + d x_3 = a x_3 + c x_4 \\ c x_2 = b x_3 \quad \longleftarrow \text{ equiv. a I} \end{cases}$$

$$\begin{cases} b x_1 - b x_4 + (d-a) x_2 = 0 \\ c x_1 - c x_4 + (d-a) x_3 = 0 \\ c x_2 - b x_3 = 0 \end{cases}$$

$$c \, \mathrm{I} - b \, \mathrm{II} = (d-a)(\mathrm{III})$$

Se $d \neq a$ lo spazio è definito da 2 eq. lineari (posso eliminare III)

$\Rightarrow$ il mio spazio è nucleo di

$$\varphi : M_{2\times 2}(\mathbb{R}) \longrightarrow \mathbb{R}^2$$

$$\varphi\begin{pmatrix} x_1 & x_2 \\ x_3 & x_4 \end{pmatrix} = \begin{pmatrix} bx_1 - bx_4 + (d-a)x_2 \\ cx_1 - cx_4 + (d-a)x_3 \end{pmatrix}$$

lo spazio è $\operatorname{Ker}\varphi$

$$\dim \operatorname{Ker}\varphi = 4 - \dim \operatorname{Im}\varphi$$

$$\dim \operatorname{Im}\varphi \leq 2 \qquad \text{perchè } \operatorname{Im}\varphi \subset \mathbb{R}^2$$

$$\Rightarrow \dim \operatorname{Ker}\varphi \geq 2$$

Se $d = a$

$$\begin{cases} b(x_1 - x_4) = 0 \\ c(x_1 - x_4) = 0 \\ cx_2 - bx_3 = 0 \end{cases}$$

$\dim = 4$ se $b = c = 0$

$\dim = 2$ altrim.