Alg. Lin. 9/12/14

Foglio 27/11.

(5b)

$$\left(\begin{array}{ccc|c} 4 & -3 & 7 & 2 \\ -3 & 5 & 1 & -1 \\ 10 & -13 & 5 & 4 \end{array}\right)$$

$$\det \uparrow = \det \begin{vmatrix} 25 & -38 & 0 \\ * & * & 1 \\ 25 & -38 & 0 \end{vmatrix} = 0$$

$\Rightarrow$ NO Cramer. $\begin{cases} \mathrm{I} \\ \mathrm{II} \\ \mathrm{III} \end{cases} \xrightarrow{\text{omogeneo}} \begin{cases} \mathrm{I}_0 \\ \mathrm{II}_0 \\ \mathrm{III}_0 \end{cases}$.

$\det(A) = 0 \Longrightarrow \mathrm{I}_0, \mathrm{II}_0, \mathrm{III}_0$ lin. dip.

ma II_0 non è $k\,\mathrm{I}_0 \Longrightarrow \mathrm{III}_0 = \alpha \cdot \mathrm{I}_0 + \beta \cdot \mathrm{II}_0$.

$\mathrm{III} \overset{?}{=} \alpha \cdot \mathrm{I} + \beta \cdot \mathrm{II}$

$\rightarrow$ No : sist. impossibile cioè $\{\text{soluz}\} = \emptyset$

$\rightarrow$ Sì : $\{\text{soluz}\} = \text{retta}$

Per noi :

$$
\left(\begin{array}{ccc|c}
4 & -3 & 7 & 2 \\
-3 & 5 & 1 & -1 \\
10 & -13 & 5 & 4
\end{array}\right)
$$

$III_0 = 1\cdot I_0 - 2\cdot II_0$ $\qquad 1\cdot 2 - 2\cdot(-1) \overset{?}{=} 4$ Sì

$\Longrightarrow III = I - 2\cdot II$

Sistema equivale a

$$
\begin{cases}
4x - 3y + 7z = 2 \\
-3x + 5y + z = -1
\end{cases}
$$

soluz:
$$\frac{1}{11}\begin{pmatrix}7\\2\\0\end{pmatrix} + \mathrm{Span}\begin{pmatrix}-38\\-25\\11\end{pmatrix}.$$

scelgo

(5c)
$$\begin{array}{ccc|c} 7 & -1 & 2 & 4 \\ 2 & 3 & -5 & -3 \\ 1 & -10 & 17 & 12 \end{array}$$

$$\det\begin{pmatrix} & \uparrow & \end{pmatrix} = \det\begin{pmatrix} 0 & 69 & -117 \\ 0 & 23 & -39 \\ 1 & * & * \end{pmatrix} = 0$$

$\Rightarrow \{soluz\} = \emptyset$ o una retta

$III_0 = 1 \cdot I_0 - 3 II_0$ $\qquad 1 \cdot 4 - 3 \cdot (-3) \overset{?}{=} 12 \qquad \underline{No}: \emptyset$

⑥ Quante soluz. ha:

$$\left(\begin{array}{ccc|c} 1-t & 2 & -2 & 1 \\ 1+t & 3 & 1 & t \\ 7 & 12 & 2t & -1 \end{array}\right) \quad ?$$

$$\det(\uparrow) = -6t^2 + 6t + 14 - 24t - 24$$

$$-4t^2 \quad -4t \quad +42$$

$$12t \quad -12$$

$$-10t^2 - 10t + 20 = -10(t^2 + t - 2)$$

$$= -10(t-1)(t+2)$$

Se $t \neq 1, -2$ ho soluz. unica.

Grazie! $\text{rank}(A) \geq 2$

$\Longrightarrow$ per $t = 1, -2$ le soluz. sono $\emptyset$ o rette.

$t = 1$

$$\left(\begin{array}{ccc|c} 0 & 2 & -2 & 1 \\ 2 & 3 & 1 & 1 \\ 7 & 12 & 2 & -1 \end{array}\right)$$

$III_0 = \frac{3}{4} \cdot I_0 + \frac{7}{2} \cdot II_0 \qquad \frac{3}{4} \cdot 1 + \frac{7}{2} \cdot 1 \overset{?}{=} -1 \qquad \underline{L_0} : \emptyset$

$t = -2$:

$$\left(\begin{array}{ccc|c} 3 & 2 & -2 & 1 \\ -1 & 3 & 1 & -2 \\ 7 & 12 & -4 & -1 \end{array}\right)$$

$III_0 = 3 \cdot I_0 + 2 \cdot II_0 \; ; \; 3 \cdot 1 + 2 \cdot (-2) \overset{?}{=} -1$ Sr

$\Longrightarrow$ {soluz} = retta (infinite soluzioni)

$$\begin{pmatrix} -3 \\ 0 \\ -5 \end{pmatrix} + \mathrm{Span}\begin{pmatrix} 8 \\ -1 \\ 11 \end{pmatrix}$$ – Non richiesto dal testo –

⑦ Calcolare i determinanti delle orlate di B in A:

$$A = \begin{pmatrix} 4 & -1 & 6 & 4 \\ -3 & 5 & 7 & 6 \\ 2 & -2 & -3 & -1 \\ 1 & 3 & 2 & 5 \end{pmatrix} \qquad B = \begin{pmatrix} -3 & 6 \\ 1 & 5 \end{pmatrix}$$

4 orlate:

$$\begin{pmatrix} 4 & -1 & 4 \\ -3 & 5 & 6 \\ 1 & 3 & 5 \end{pmatrix} \quad \begin{pmatrix} 4 & 6 & 4 \\ -3 & 7 & 6 \\ 1 & 2 & 5 \end{pmatrix}$$

$$\begin{pmatrix} -3 & 5 & 6 \\ 2 & -2 & -1 \\ 1 & 3 & 5 \end{pmatrix} \quad \begin{pmatrix} -3 & 7 & 6 \\ 2 & -3 & -1 \\ 1 & 2 & 5 \end{pmatrix}$$

da calcolare i det.

⑧ Calcolare rank via orlati:

(a) $\begin{pmatrix} 2 & 1 & 5 & 5 \\ 1 & 2 & 1 & 7 \\ 3 & 1 & 8 & 6 \end{pmatrix}$ $\Rightarrow$ rank ≥ 2

$$\det\begin{pmatrix} 2 & 1 & 5 \\ 1 & 2 & 1 \\ 3 & 1 & 8 \end{pmatrix} = 32+3+5-30-8-2 = 0 \quad \left(\underline{\mathrm{III}} : 3\cdot \mathrm{I} - \mathrm{II}\right)$$

$$\det\begin{pmatrix} 2 & 1 & 5 \\ 1 & 2 & 7 \\ 3 & 1 & 6 \end{pmatrix} = 24+21+5-30-6-14 = 0 \quad \left(\mathrm{III} = \mathrm{I} + 3\cdot \mathrm{II}\right)$$

$$\Rightarrow \text{rank} = 2$$

(b)

$$\begin{array}{c} \begin{pmatrix} 0 & 0 & 1 & 1 & 2 \\ 1 & -2 & 0 & 1 & 0 \\ 3 & -6 & 1 & 4 & 3 \end{pmatrix} \\ \checkmark \quad\ \times \quad\ \checkmark \quad\ \times \quad\ \checkmark \end{array} \qquad \Rightarrow \text{rank} = 3$$

(c) $\begin{pmatrix} 2 & 1 & 5 & 4 \\ 1 & 3 & 5 & 7 \\ 0 & 1 & 1 & 2 \\ 1 & 0 & 2 & 1 \end{pmatrix}$ rank ≥ 2

$$\det(A) = \begin{vmatrix} 0 & 1 & 1 & 2 \\ 0 & 3 & 3 & 6 \\ 0 & 1 & 1 & 2 \\ 1 & * & * & * \end{vmatrix} = 0$$

Anzi: $\mathrm{rank}(A) = \mathrm{rank}(\quad) = 1 + 1 = 2$.

(Facendo i 4 det 3×3 avrei trovato sempre 0) _

⑨ Come può cambiare il rango di una matrice cambiandone un solo coeff?

Anzi: come può cambiare il rango cambiando una sola colonna? (La prima)

$$A = (v_1, v_2, \dots, v_n) \rightsquigarrow (v_1', v_2, \dots, v_n) = A'$$

Se $\dim\big(\overbrace{\mathrm{Span}(v_2, \dots, v_n)}^{W}\big) = k$

$$\mathrm{rank}(A) = \begin{cases} k & \text{se } v_1 \in \bar{W} \\ k+1 & \text{se } v_1 \notin \bar{W} \end{cases} ; \quad \mathrm{rank}(A') = \begin{cases} k & v_1' \in \bar{W} \\ k+1 & v_1' \notin \bar{W} \end{cases}$$

$\Rightarrow$ il rank può restare uguale

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \longrightarrow \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$$

- aumentare/calare di 1 : $\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \longleftrightarrow \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

(10) Calcolare $\dim(E)$ e passare da equaz. cart. a param. o viceversa.

(a) $\mathbb{R}^2$; $6x-5y=11$; $\dim = 2-1 = 1$.

(cart.)

param: $\begin{pmatrix} 1 \\ -1 \end{pmatrix} + \mathrm{Span}\begin{pmatrix} 5 \\ 6 \end{pmatrix}$ $\circledast$

OK

$\begin{pmatrix} 11/6 \\ 0 \end{pmatrix} + \mathrm{Span}\begin{pmatrix} 5\sqrt{17\pi} \\ 6\sqrt{17\pi} \end{pmatrix}$ giusto!

$\otimes = \begin{cases} x = 1+5t \\ y = -1+6t \end{cases}$ ok $= \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} + t\begin{pmatrix} 5 \\ 6 \end{pmatrix}$ ok

$$\overbrace{= \left\{ \begin{pmatrix} x \\ y \end{pmatrix} + t\begin{pmatrix} 5 \\ 6 \end{pmatrix} : t \in \mathbb{R} \right\}}^{ok}$$

(b) $\mathbb{R}^2$; $E : \begin{cases} x = -7+2t \\ y = 5+6t \end{cases}$

$$E = \left\{ \begin{pmatrix} -7 \\ 5 \end{pmatrix} + t\begin{pmatrix} 2 \\ 6 \end{pmatrix} : t \in \mathbb{R} \right\}$$

$$= \left\{ \begin{pmatrix} -7 \\ 5 \end{pmatrix} + t\begin{pmatrix} 1 \\ 3 \end{pmatrix} : t \in \mathbb{R} \right\} \quad \text{(param)}$$

$\dim = 1$.

cart : $3x - y = -36$

(c) $E \subset \mathbb{R}^3$; $E : \begin{cases} 2x - 3y + 7z = 6 \\ -5x + y + 6z = 2 \end{cases}$ (cart.)

$\dim = 3 - 2 = 1$.

param : $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \mathrm{Span} \begin{pmatrix} +25 \\ +47 \\ +13 \end{pmatrix}$.

(d) $E \subset \mathbb{R}^3$: E : $\begin{cases} x = 4 - 2t \\ y = -5 + t \\ z = 3 + 5t \end{cases}$ (param)

$E = \begin{pmatrix} 4 \\ -5 \\ 3 \end{pmatrix} + \text{Span}\begin{pmatrix} -2 \\ 1 \\ 5 \end{pmatrix}$ dim = 1

cart. $\begin{cases} x + 2y = -6 \\ 5x + 2z = 26 \end{cases}$

(e) $E \subset \mathbb{R}^3$: E : $5x - 12y + 10z = 2$ cart

$$E = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3 : 5x - 12y + 10z = 2 \right\}.$$

$$\dim = 3 - 1 = 2$$

param : $\begin{pmatrix} 0 \\ -1 \\ -1 \end{pmatrix} + \mathrm{Span}\left(\begin{pmatrix} 12 \\ 5 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 5 \\ 6 \end{pmatrix} \right)$

$\begin{pmatrix} 12 \\ 4 \\ -1 \end{pmatrix} + \mathrm{Span}\left(\begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 12\pi \\ 5\pi + 5e \\ 6e \end{pmatrix} \right) \leftarrow$ giusto.

(1) $E \subset \mathbb{R}^3$ $\quad E: \begin{cases} x = 3 + 2t - 5s \\ y = 6 + 5t - 3s \\ z = -7 + 4t - 7s \end{cases}$ param.

$$E = \begin{pmatrix} 3 \\ 6 \\ -7 \end{pmatrix} + \mathrm{Span}\left(\begin{pmatrix} 2 \\ 5 \\ 4 \end{pmatrix}, \begin{pmatrix} 5 \\ 3 \\ 7 \end{pmatrix} \right) \qquad \dim = 2$$

cart: $\quad 23x + 6y - 19z = 238$

$$\begin{array}{r} 69 \\ 36 \\ 133 \\ \hline 238 \end{array}$$

(g) $E \subset \mathbb{R}^3$: $E: \begin{cases} 2x + 3y - 4z = 1 \\ 3x - 2y + 5z = -1 \\ \cancel{7x + 4y - 3z = 1} \end{cases}$

$\dim E \overset{?}{=} 3 - 3$

solo se sono lin. indip.

Invece: $\mathrm{III} = 2 \cdot \mathrm{I} + \mathrm{II}$ $\Rightarrow \dim = 1$

$$\Rightarrow \begin{pmatrix} \vdots \end{pmatrix} + \mathrm{Span} \begin{pmatrix} 7 \\ -22 \\ -13 \end{pmatrix}.$$