Geometria 28/3/14

(18) Base ortog. di:

$$\mathrm{Span}\left(\begin{pmatrix}3\\1\\-5\\2\end{pmatrix}, \begin{pmatrix}-2\\4\\3\\-1\end{pmatrix}\right)^{\perp} = \mathrm{Span}\left(\begin{pmatrix}23\\-5\\14\\0\end{pmatrix} \begin{pmatrix}0\\-1\\9\\23\end{pmatrix}\right)$$

Base ortog: $\begin{pmatrix}0\\-1\\9\\23\end{pmatrix}, \begin{pmatrix}23\\-5\\14\\0\end{pmatrix} - \dfrac{5+14\cdot 9}{1+81+23^2}\begin{pmatrix}0\\-1\\9\\23\end{pmatrix} = \ldots$

(20) Calcolare $H_0 f$ verificando che è simmetrica.

(b) $f(x,y) = y \cdot \cos(x-3y^2) - 2x \sin(y+5x^2)$

$$\frac{\partial f}{\partial x} = -y \sin(x-3y^2) - 2\sin(y+5x^2) - 20x^2 \cos(y+5x^2)$$

$$\frac{\partial f}{\partial y} = \cos(x-3y^2) + 6y^2 \sin(x-3y^2) - 2x\cos(y+5x^2)$$

$$\frac{\partial^2 f}{\partial x^2} = -y\cos(x-3y^2) - 20x\cos(y+5x^2) + 40x\cos(y+5x^2) + 200x^3 \sin(y+5x^2)$$

$$\frac{\partial^2 f}{\partial y \partial x} = -\sin(x-3y^2) + 6y^2\cos(x-3y^2) - 2\cos(y+5x^2) + 20x^2\sin(y+5x^2)$$

$$\frac{\partial^2 f}{\partial x \partial y} = -\sin(x-3y^2) + 6y^2\cos(x-3y^2) - 2\cos(y+5x^2) + 20x^2\sin(y+5x^2)$$

$$\frac{\partial^2 f}{\partial y^2} = \dots \qquad H_0 f = \begin{pmatrix} 0 & -2 \\ -2 & 0 \end{pmatrix}$$

(21) Dato V con $\langle \cdot | \cdot \rangle$ scrivere la proiez. $\perp$ su W.

(a) $\mathbb{R}^2$, $\langle \cdot | \cdot \rangle_{\mathbb{R}^2}$ $\quad W = \{x : 4x_1 - 3x_2 = 0\}$.

$$W = \mathrm{Span}\begin{pmatrix} 3 \\ 4 \end{pmatrix} ; \quad P_W(x) = \frac{3x_1 + 4x_2}{5}\begin{pmatrix} 3 \\ 4 \end{pmatrix} = \frac{1}{5}\begin{pmatrix} 9 & 12 \\ 12 & 16 \end{pmatrix} \cdot x$$

(b) $\mathbb{R}^2$, $\langle \cdot | \cdot \rangle_{\begin{pmatrix} 5 & -2 \\ -2 & 1 \end{pmatrix}}$ $\quad W = \mathrm{Span}\begin{pmatrix} 3 \\ 4 \end{pmatrix}$

$$p_W(x) = \frac{5 \cdot x_1 3 - 2(x_1 \cdot 4 + x_2 \cdot 3) + 1 \cdot x_2 \cdot 4}{5 \cdot 3^2 - 4 \cdot 3 \cdot 4 + 4^2} \begin{pmatrix} 3 \\ 4 \end{pmatrix} = M \cdot x$$

Siccome p_W è proiez. avrò $M \cdot M = M$; inoltre se $A = \begin{pmatrix} 5 & -2 \\ -2 & 1 \end{pmatrix}$ avrò che M è autoaggiunta rispetto ad $\langle \cdot | \cdot \rangle_A$, cioè:

$$\langle M \cdot v | w \rangle_A = \langle v | M \cdot w \rangle_A \qquad \forall v, w$$

$$ {}^t v \cdot {}^t M \cdot A \cdot w = {}^t v \cdot A \cdot M \cdot w \qquad \forall v, w $$

$$ {}^t M \cdot A = A \cdot M $$

Condizione di autoaggiunzione di M rispetto a $\langle \cdot | \cdot \rangle_A$.

(Esercizio: verificarlo.)

(c) $\mathbb{R}^3$, $\langle \cdot | \cdot \rangle_{\mathbb{R}^3}$, $W = \{ x : x_1 - 2x_2 + 5x_3 = 0 \}$.

Come non risolverlo: $w_1 = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} \quad w_2 = \begin{pmatrix} 0 \\ 5 \\ 2 \end{pmatrix}$

ortonormalizzo $u_1 = \frac{1}{\sqrt{5}} \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}, \quad u_2 = \frac{\begin{pmatrix} 0 \\ 5 \\ 2 \end{pmatrix} - \dots}{\| \quad '' \quad \|}$

$$p_W(x) = \langle x | u_1 \rangle \cdot u_1 + \langle x | u_2 \rangle \cdot u_2$$

Invece: $W = \begin{pmatrix} 1 \\ -2 \\ 5 \end{pmatrix}^{\perp} \Longrightarrow W^{\perp} = \operatorname{Span}\begin{pmatrix} 1 \\ -2 \\ 5 \end{pmatrix}$

$$p_W(x) = x - p_{W^\perp}(x) = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} - \frac{x_1 - 2x_2 + 5x_3}{1 + 4 + 25} \begin{pmatrix} 1 \\ -2 \\ 5 \end{pmatrix}$$

$$= \underbrace{\frac{1}{30}\begin{pmatrix} 29 & 2 & -5 \\ 2 & 26 & 10 \\ -5 & 10 & 5 \end{pmatrix}}_{M} \cdot x$$

$${}^tM = M \quad \checkmark$$

$$M \cdot M = M \quad \ldots$$

(d) $\mathbb{R}^3, \ \langle \cdot | \cdot \rangle_{\mathbb{R}^3} \qquad W = \begin{cases} 3x_1 + 2x_2 = 0 \\ 3x_2 - 5x_3 = 0 \end{cases}$

$$W = \left(\begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 3 \\ -5 \end{pmatrix} \right)^{\perp} = \mathrm{Span}\left(\begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix} \wedge \begin{pmatrix} 0 \\ 3 \\ -5 \end{pmatrix} \right) = \mathrm{Span}\begin{pmatrix} -10 \\ 15 \\ 9 \end{pmatrix}$$

$$p_W(x) = \frac{-10x_1 + 15x_2 + 9x_3}{100 + 225 + 81} \begin{pmatrix} -10 \\ 15 \\ 9 \end{pmatrix}$$

(e) $\mathbb{R}^3, \langle \cdot | \cdot \rangle_A \qquad A = \begin{pmatrix} 1 & 2 & 0 \\ 2 & 5 & -1 \\ 0 & -1 & 3 \end{pmatrix} \qquad W: x_1 - 2x_2 + 5x_3 = 0$

$(d_1 = 1 > 0,\ d_2 = 1 > 0,\ d_3 = 15 - 12 - 1 > 0 : \checkmark)$

$W = \mathrm{Span}\begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 5 \\ 2 \end{pmatrix}$ → ortogonalizzo . . .

→ trovo ortogonale . . . QUESTO

Cerco un generatore y di $W^\perp$:

$$y \cdot \begin{pmatrix} 1 & 2 & 0 \\ 2 & 5 & -1 \\ 0 & -1 & 3 \end{pmatrix}\begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} = 0 \qquad y \cdot \begin{pmatrix} 1 & 2 & 0 \\ 2 & 5 & -1 \\ 0 & -1 & 3 \end{pmatrix}\begin{pmatrix} 0 \\ 5 \\ 2 \end{pmatrix} = 0$$

$$4y_1 + 9y_2 - y_3 = 0 \qquad 10y_1 + 23y_2 + y_3 = 0$$

$$y = \begin{pmatrix} 4 \\ 9 \\ -1 \end{pmatrix} \wedge \begin{pmatrix} 10 \\ 23 \\ 1 \end{pmatrix} = \begin{pmatrix} 32 \\ -14 \\ 2 \end{pmatrix} \rightsquigarrow y = \begin{pmatrix} 16 \\ -7 \\ 1 \end{pmatrix}$$

$$\left(64 - 63 - 1 = 0 \ \checkmark \qquad 160 - 161 + 1 = 0 \ \checkmark \right)$$

$$P_W(x) = x - P_{W^\perp}(x) = x - \frac{\langle x | y \rangle_A}{\|y\|_A^2} \cdot y$$

$$y = \begin{pmatrix} 16 \\ -7 \\ 1 \end{pmatrix} \qquad A = \begin{pmatrix} 1 & 2 & 0 \\ 2 & 5 & -1 \\ 0 & -1 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} - \frac{1 \cdot 16 \cdot x_1 + 5 \cdot (-7) \cdot x_2 + 3 \cdot 1 \cdot x_3 + 2 \cdot (16 x_2 - 7 x_1) - (-7 x_3 + x_2)}{1 \cdot 16^2 + 5 \cdot (-7)^2 + 3 \cdot 1^2 + 4 \cdot 16 \cdot (-7) - 2 \cdot (-7) \cdot 1} \cdot \begin{pmatrix} 16 \\ -7 \\ 1 \end{pmatrix}$$

$= M \cdot x$. Verificare $M \cdot M = M$ e ${}^tM \cdot A = A \cdot M$

(22) Trovare eq. par. di $\underline{P}^{\perp}$ per $\underline{P}$ piano dato in $\mathbb{R}^3$:

$$\underline{P} = \operatorname{Span}\left(\begin{pmatrix}5\\4\\-1\end{pmatrix}\begin{pmatrix}-3\\2\\7\end{pmatrix}\right) \Rightarrow \underline{P}^{\perp} = \operatorname{Span}\begin{pmatrix}5\\4\\-1\end{pmatrix}\wedge\begin{pmatrix}-3\\2\\7\end{pmatrix}$$

$$= \operatorname{Span}\ \cancel{\begin{pmatrix}30\\-32\\22\end{pmatrix}}\ \begin{pmatrix}15\\-16\\11\end{pmatrix}$$

$75-64-11=0$ ✓ $\quad -45-32+77=0$ ✓

(23) Trovare eq. cart. di $l^\perp$ con $l \subset \mathbb{R}^3$ retta data

$$l : \begin{cases} 2x + 5y - z = 0 \\ 6x - 7y + z = 0 \end{cases}$$

$$l = \left(\operatorname{Span}\left(\begin{pmatrix} 2 \\ 5 \\ -1 \end{pmatrix}, \begin{pmatrix} 6 \\ -7 \\ 1 \end{pmatrix} \right) \right)^\perp$$

$$\Longrightarrow l^\perp = \operatorname{Span}\left(\begin{pmatrix} 2 \\ 5 \\ -1 \end{pmatrix}, \begin{pmatrix} 6 \\ -7 \\ 1 \end{pmatrix} \right)$$

$$= \left(\begin{pmatrix} 2 \\ 5 \\ -1 \end{pmatrix} \wedge \begin{pmatrix} 6 \\ -7 \\ 1 \end{pmatrix} \right)^\perp = \begin{pmatrix} -2 \\ -8 \\ -44 \end{pmatrix}^\perp = \begin{pmatrix} 1 \\ 4 \\ 22 \end{pmatrix}^\perp$$

$\Rightarrow \ell^{\perp}: \quad x + 4y + 22 \cdot z = 0$

$\left(2 + 20 - 22 = 0 \ \checkmark \qquad 6 - 28 + 22 = 0 \ \checkmark \right)$