

Esercizi di Algebra Lineare (Petronio 12/13)

5 ottobre 2012

Nello spazio vettoriale V assegnato considerare l'insieme W descritto e stabilire se W sia un sottospazio vettoriale di V .

Esercizio 1 $V = \mathbb{R}^n$

$$(a) \quad W = \left\{ x : \sum_{j=1}^n \sqrt{j-1} \cdot x_j = 0 \right\}$$

$$(b) \quad W = \left\{ x : \sum_{j=1}^n x_j^j = 0 \right\}$$

$$(c) \quad W = \{x : x_1^2 = x_2^2\}$$

$$(d) \quad W = \{x : x_1^3 = x_2^3\}$$

$$(e) \quad W = \{x : \cos(x_1 + \dots + x_n) = 1\}$$

Esercizio 2. $V = \mathcal{M}_{m \times n}(\mathbb{R})$

$$(a) \quad W = \{A : (A)_{1,1} \cdot (A)_{m,n} = 0\}$$

$$(b) \quad W = \left\{ A : \sum_{j=1}^{\min\{m,n\}} (A)_{j,n+1-j} = 0 \right\}$$

$$(c) \quad W = \left\{ A : \sum_{i=1}^m \sum_{j=1}^n \frac{\sqrt{i}-\sqrt{j}}{\sqrt{i}+\sqrt{j}} \cdot (A)_{i,j} = 0 \right\}$$

$$(d) \quad W = \left\{ A : \sum_{j=1}^{\min\{m,n\}} (A)_{j,j}^2 = 0 \right\}$$

$$(e) \quad W = \{A : |(A)_{i,j}| \leq 7 \text{ per } 1 \leq i \leq m, 1 \leq j \leq n\}$$

Esercizio 3. $V = \mathbb{R}[t]$

$$(a) \quad \{p(t) : p(-3) + p''(2) = 0\}$$

$$(b) \quad \{p(t) : p(1) \cdot p'''(-1) = 0\}$$

$$(c) \quad \left\{ p(t) : \sum_{n=0}^{+\infty} (-1)^n \cdot t^n \cdot p^{(n)}(t) = 0 \right\}$$

$$(d) \quad \{p(t) : |p(t)| \leq 1 + e^t \quad \forall t \in \mathbb{R}\}$$

$$(e) \quad \{p(t) : \deg(2p'(t) - 3t^2p'''(t)) \leq 5\}$$

Esercizio 4. $V = \mathcal{F}(\{a, b, c\}, \mathbb{R})$

$$(a) \quad W = \{f : f(c) \geq 2f(a)\}$$

$$(b) \quad W = \{f : 3f(a) - 7f(b) = 0\}$$

$$(c) \quad W = \{f : f(a) \cdot f(b) + f(a) \cdot f(c) + f(b) \cdot f(c) = 0\}$$

$$(d) \quad W = \{f : \ln(1 + |4f(a) - 3f(c)|) = 0\}$$

$$(e) \quad W = \{f : |f(b) + f(c)| \leq |f(a)|\}$$



Nello spazio vettoriale V dato considerare i vettori $v, w_1, \dots, w_k$ indicati, stabilire se v appartenga al sottospazio $W = \text{Span}(w_1, \dots, w_k)$ e in tal caso dire se l'espressione di v come combinazione lineare di $w_1, \dots, w_k$ sia unica.

Esercizio 5. $V = \mathbb{R}^2$

$$(a) \ v = \begin{pmatrix} 3 \\ -4 \end{pmatrix}, w_1 = \begin{pmatrix} -2 \\ 5 \end{pmatrix}$$

$$(b) \ v = \begin{pmatrix} \sqrt{6} \\ -2 \end{pmatrix}, w_1 = \begin{pmatrix} -3 \\ \sqrt{6} \end{pmatrix}$$

$$(c) \ v = \begin{pmatrix} 3 \\ 7 \end{pmatrix}, w_1 = \begin{pmatrix} 4 \\ -11 \end{pmatrix}, w_2 = \begin{pmatrix} 3 \\ 8 \end{pmatrix}$$

$$(d) \ v = \begin{pmatrix} -2 \\ 7 \end{pmatrix}, w_1 = \begin{pmatrix} 4 \\ 1 \end{pmatrix}, w_2 = \begin{pmatrix} 2 \\ -1 \end{pmatrix}, w_3 = \begin{pmatrix} 5 \\ -3 \end{pmatrix}$$

Esercizio 6. $V = \mathbb{R}^3$

$$(a) \ v = \begin{pmatrix} 4 \\ 5 \\ -2 \end{pmatrix}, w_1 = \begin{pmatrix} 3 \\ -6 \\ 1 \end{pmatrix}$$

$$(b) \ v = \begin{pmatrix} 2 \\ -5 \\ 4 \end{pmatrix}, w_1 = \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix}, w_2 = \begin{pmatrix} 4 \\ 1 \\ -2 \end{pmatrix}$$

$$(c) \ v = \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix}, w_1 = \begin{pmatrix} 5 \\ 1 \\ 2 \end{pmatrix}, w_2 = \begin{pmatrix} -2 \\ 2 \\ 3 \end{pmatrix}$$

$$(d) \ v = \begin{pmatrix} -1 \\ 3 \\ -8 \end{pmatrix}, w_1 = \begin{pmatrix} 2 \\ 2 \\ -3 \end{pmatrix}, w_2 = \begin{pmatrix} 5 \\ 1 \\ 2 \end{pmatrix}, w_3 = \begin{pmatrix} 3 \\ -1 \\ 5 \end{pmatrix}$$

$$(e) \ v = \begin{pmatrix} -4 \\ 1 \\ 2 \end{pmatrix}, w_1 = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}, w_2 = \begin{pmatrix} -1 \\ 0 \\ 3 \end{pmatrix}, w_3 = \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}$$

$$(f) \quad v = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, w_1 = \begin{pmatrix} 4 \\ 1 \\ 0 \end{pmatrix}, w_2 = \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}, w_3 = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}, w_4 = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$$

Esercizio 7. $V = \mathbb{R}[t]$

$$(a) \quad v = 1 - 2t + 3t^2, w_1 = 4 + t - t^2, w_2 = -3 + 2t + t^2$$

$$(b) \quad v = 4 - 3t + t^2, w_1 = 2 + t - t^2, w_2 = 2 - 9t + 5t^2$$

$$(c) \quad v = 7 + t - t^2, w_1 = 5 - t + 2t^2, w_2 = -2 - 2t + 3t^2, w_3 = 1 - 5t + 8t^2$$

$$(d) \quad v = 2t^2, w_1 = 1 + t, w_2 = 1 - t + t^2, w_3 = 3t - t^2$$

Esercizio 8. $V = \mathcal{M}_{2 \times 2}(\mathbb{R})$

$$(a) \quad v = \begin{pmatrix} 2 & 3 \\ -1 & 7 \end{pmatrix}, w_1 = \begin{pmatrix} 2 & 0 \\ 1 & -1 \end{pmatrix}, w_2 = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}, w_3 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix},$$

$$(b) \quad v = \begin{pmatrix} 5 & -1 \\ 2 & 1 \end{pmatrix}, w_1 = \begin{pmatrix} 7 & 1 \\ -1 & 2 \end{pmatrix}, w_2 = \begin{pmatrix} 1 & 2 \\ 3 & -1 \end{pmatrix}, w_3 = \begin{pmatrix} 1 & 0 \\ -6 & 2 \end{pmatrix},$$

$$(c) \quad v = \begin{pmatrix} 3 & 0 \\ 3 & 4 \end{pmatrix}, \\ w_1 = \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix}, w_2 = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}, w_3 = \begin{pmatrix} 3 & 1 \\ 1 & 4 \end{pmatrix}, w_4 = \begin{pmatrix} 0 & -1 \\ 2 & 0 \end{pmatrix}$$

$$(c) \quad v = \begin{pmatrix} 4 & 1 \\ -3 & 2 \end{pmatrix}, \\ w_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, w_2 = \begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}, w_3 = \begin{pmatrix} 0 & -1 \\ 3 & 0 \end{pmatrix}, w_4 = \begin{pmatrix} 0 & 0 \\ -1 & 1 \end{pmatrix}$$



Esercizio 9. Nello spazio vettoriale V dato considerare il sottospazio vettoriale W e i vettori $w_1, \dots, w_k$ indicati. Usando la definizione stabilire se $w_1, \dots, w_k$ costituiscano un sistema di generatori per W , e in caso affermativo se ogni $w \in W$ abbia un'unica espressione come combinazione lineare di $w_1, \dots, w_k$.

$$(a) \quad V = \mathbb{R}^3, W = \{x : x_1 + 3x_2 - 2x_3 = 0\}, \\ w_1 = \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}, w_2 = \begin{pmatrix} 3 \\ -1 \\ 0 \end{pmatrix}, w_3 = \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix}$$

$$(b) \quad V = \mathbb{R}^3, W = \{x : 5x_1 + 2x_2 - x_3 = 0\}, w_1 = \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}, w_2 = \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix}$$

$$(c) \quad V = \mathbb{R}^3, W = \{x : 3x_1 - 2x_2 + x_3 = 0\}, w_1 = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}, w_2 = \begin{pmatrix} 3 \\ 5 \\ 1 \end{pmatrix}$$

$$(d) \quad V = \mathbb{R}^3, W = \{x : 4x_1 + 3x_2 - 5x_3 = 0\}, \\ w_1 = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}, w_2 = \begin{pmatrix} 1 \\ -3 \\ -1 \end{pmatrix}, w_3 = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$$

$$(e) \quad V = \mathbb{R}^4, W = \{x : x_1 + 3x_2 + x_3 - x_4 = 0\}, w_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 5 \end{pmatrix}, w_2 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ 0 \end{pmatrix}$$

$$(f) \quad V = \mathbb{R}^4, W = \{x : x_3 = 2x_1 + x_2, x_4 = x_1 - 3x_2\}, \\ w_1 = \begin{pmatrix} 3 \\ -4 \\ 2 \\ 15 \end{pmatrix}, w_2 = \begin{pmatrix} -1 \\ 2 \\ 0 \\ -7 \end{pmatrix}$$

$$(g) \quad V = \mathbb{R}^4, W = \{x : x_3 = x_1 - 2x_2, x_4 = 3x_1 + 2x_2\}, \\ w_1 = \begin{pmatrix} 1 \\ 2 \\ -3 \\ 7 \end{pmatrix}, w_2 = \begin{pmatrix} 3 \\ 1 \\ 1 \\ 11 \end{pmatrix}, w_3 = \begin{pmatrix} 2 \\ -1 \\ 4 \\ 4 \end{pmatrix}$$