Curriculum Vitæ et Studiorum
Carlo Petronio

January 2, 2007

Contents

1 Titles

March 31, 1995. PhD degree *cum laude* from the Scuola Normale Superiore di Pisa. Dissertation entitled *Standard spines and* 3*-manifolds* supervised by Riccardo Benedetti. Evaluation committee: Riccardo Benedetti, Michel Boileau, Corrado de Concini, Maria Dedò, Margherita Galbiati, Hugh Morton, Edoardo Vesentini.

April 23, 1994. Francesco Severi prize.

July 13, 1989. First degree in Mathematics *cum laude* from the University of Pisa; dissertation entitled *Variations on a theme of Thullen: non-homogeneous bounded domains in Banach spaces*, supervised by Edoardo Vesentini.

July 13, 1989. Diploma from the Scuola Normale Superiore di Pisa.

2 Positions

2002/03 – today. Full professor of geometry at the Faculty of Engineering, University of Pisa.

1992 – today. Secretary of the Editorial Board, Annali della Classe di Scienze, Scuola Normale Superiore di Pisa.

February-March 2007. Invited professor at the University of Toulouse.

May 2003. Invited professor at the University of Paris VII.

1999/00 – 2001/02. Associate professor of geometry at the Faculty of Engineering, University of Pisa.

1993/94 – 1998/99. Research assistant at the Faculty of Science, University of Pisa.

1992/93. Civil service (as a conscientious objector to the military service).

1991/92. Visiting graduate student at the Mathematics Institute of the University of Warwick (UK).

1989/90 – 1991/92. Graduate student of Mathematics at the Scuola Normale Superiore di Pisa.

1985/86 – 1988/89. Student of Mathematics, Scuola Normale Superiore di Pisa and University of Pisa.

3 Teaching

The lists of topics covered in all my courses starting from 1998/99, together with support material for students and the solutions of all exam problems, are available from my web site:

`http://www.dm.unipi.it/pages/petronio/public_html/`

2005/2006.

- “Mathematics III,” third year, degree in Telecommunications Engineering, University of Pisa (100 hours).
- “Linear algebra,” first year, degree in Telecommunications Engineering, University of Pisa (50 hours).

2004/2005.

- “Mathematics III,” third year, degree in Telecommunications Engineering, University of Pisa (100 hours).
- “Linear algebra,” first year, degree in Telecommunications Engineering, University of Pisa (50 hours).
- “Curves and surfaces” honor course for students in Engineering, University of Pisa (24 hours).

2003/2004.

- “Mathematics III,” third year, degree in Telecommunications Engineering, University of Pisa (100 hours).
- “Linear algebra,” first year, degree in Telecommunications Engineering, University of Pisa (50 hours).
- “Triangulations, handles, and normal surfaces,” doctorate school in mathematics, University of Pisa (20 hours).

2002/2003.

- “Mathematics III,” third year, degree in Telecommunications Engineering, University of Pisa (100 hours).
- “Geometry and algebra,” first year, degree in Telecommunications Engineering, University of Pisa (55 hours).

2001/2002.

- “Mathematics III,” third year, degree in Telecommunications Engineering, University of Pisa (100 hours).
- “Geometry and algebra,” first year, degree in Telecommunications Engineering, University of Pisa (55 hours).

2000/2001.

- “Mathematics III,” third year, degree in Telecommunications Engineering, University of Pisa (100 hours).
- “Geometry and algebra,” first year, degree in Telecommunications Engineering, University of Pisa (55 hours).
- “Complexity and hyperbolic geometry in dimension 3,” doctorate school in Mathematics, University of Pisa (40 hours).

1999/2000.

- “Mathematics,” third year, degree in Telecommunications Engineering, University of Pisa (100 hours).
- “Geometry and algebra,” first year, degree in Telecommunications Engineering, University of Pisa (55 hours).

1998/1999.

- “Foundations of higher geometry I,” third year, degree in Mathematics, University of Parma (60 hours).
- “Higher geometry II,” fourth year, degree in Mathematics, University of Pisa (course on the interactions between curvature and topology in Riemannian geometry, 40 hours).
- “Geometry and algebra,” first year, degree in Telecommunications Engineering, University of Pisa (100 hours).

1997/1998.

- “Geometry II,” second year, degree in Mathematics, University of Parma (120 hours).
- “Differential Geometry II,” fourth year, degree in Mathematics, University of Pisa (course on the geometric theory of foliations, 40 hours).

1995/1996.

- “Principal bundles and spin structures,” doctorate school in Mathematics, University of Pisa (20 hours).

1989/90 – 1998/99. As a graduate student and research assistant I have performed every year support teaching activities for first and second year courses in geometry at the Faculty of Science of the University of Pisa.

4 Dissertations supervised

PhD dissertations:

- Fiontan Roukema, to start in 2007.
- Maria Antonietta Pascali, *Matveev complexity and coverings of (Seifert)* 3*-manifolds*, started in 2006, in progress.
- Roberto Frigerio, *Hyperbolic* 3*-manifolds with geodesic boundary*, started in 2001 and defended in March 2005.
- Gennaro Amendola, *Minimal spines and skeleta of non-orientable* 3*-manifolds and bricks*, started in 2000 and defended in January 2004.
- Stefano Francaviglia, *Hyperbolicity equations for cusped* 3*-manifolds and volume-rigidity of representations*, started in 1999 and defended in May 2004.
- Bruno Martelli, *Decomposition into bricks and complexity of* 3*-manifolds*, started in 2000 and defended in February 2002.

Degree dissertations:

- Vincenzo De Risi, *Geometrization of* 3*-manifolds*, 2006, approved.
- Tiziano Casavecchia, *Finite and ideal hyperbolic polyhedra*, 2005, approved *cum laude.*
- Simone Penzavalle, 2003, *Heegaard splittings of Seifert fibred spaces*, approved *cum laude.*
- Ramona Bassi, 2003, *Two-dimensional orbifolds and the Riemann-Hurwitz formula*, approved.
- Paola Boito, 2001, *Coverings of* 3*-manifolds*, approved *cum laude.*
- Roberto Frigerio, 2000, *Canonical decomposition of hyperbolic* 3*-manifolds with geodesic boundary*, approved *cum laude.*
- Lucio Bedulli, 1999, *Euler structures and Reidemeister torsion*, approved *cum laude.*
- Gennaro Amendola, 1999, *Presentations and calculi in dimension three*, approved *cum laude.*
- Francesco Costantino, 1999, *Canonical decomposition of* 3*-dimensional manifolds*, approved *cum laude.*
- Marina Pescini, 1997, *Ideal cellularizations of hyperbolic* 3*-manifolds*, approved *cum laude.*

- Bruno Martelli, 1996, *Turaev-Viro invariants of* 3*-manifolds*, approved *cum laude.*

Short (non-research) dissertations:

- Daniele Calbi (Faculty of Engineering), *Lebesgue integral and distributions*, in progress.
- Marco Messina (Faculty of Engineering), *Surfaces, curvature, and the Gauss-Bonnet formula*, 2006, approved.
- Stefano Fortunati (Faculty of Engineering), 2005, *Topological surfaces and the coloring problem*, 2006, approved.
- Paola Boito, 2000, *The word problem in the braid group*, approved.
- Sara Biagini, 1999, *Two-dimensional orbifolds*, approved *cum laude.*
- Francesco Costantino, 1998, *Parallelizability of orientable* 3*-manifolds*, approved;
- Francesco Morandin, 1998, *The Kauffman bracket*, approved *cum laude.*
- Nicola Visciglia, 1997, *Tessellations of* 3*-space by knotted tori*, approved *cum laude.*

5 Invited talks and advanced courses

June 2007. After an invitation of Andrei Vesnin, two-weeks visit at the Sobolev Institute of Mathematics, Novosibirsk, and two talks on low-dimensional topology.

April 2007. After an invitation of Dylan Thurston, one-week visit at the Columbia University, New York City, and two talks on hyperbolic geometry and combinatorial methods in low-dimensional topology.

March 8, 2007. Colloquium talk at the University of Genova on the Poincaré conjecture.

February-March 2007. Series of 5 two-hours lectures at the University of Toulouse on combinatorial approaches to the topology and hyperbolic geometry of knots, graphs, 3-manifolds and orbifolds.

February 25-28, 2007. After an invitation of Wolfgang Metzler, one-hour talk at the University of Frankfurt on hyperbolic knots and graphs.

December 13, 2006. After an invitation of Corrado de Concini, one-hour seminar at the University of Rome I on enumeration of hyperbolic graphs of small complexity.

November 9-23, 2006. After an invitation of Craig Hodgson, two-week visit to Melbourne and one-hour seminar on asymptotic bounds for the complexity of hyperbolic 3-manifolds and groups.

October 14-21, 2006. After an invitation of Athanase Papadopoulos, one-week visit to Strasbourg and one-hour seminar on complexity of hyperbolic graphs.

September 19-22, 2006. Plenary speaker in the conference *Mathematics, Mechanics and Computer Science*, Chelyabinsk (Russia). One-hour talk on hyperbolic knots and graphs.

August 31-September 2, 2006. Invitation to give a one-hour talk on enumeration of knot complexity in the conference *Geometry and Topology of Low Dimensional Manifolds*, Burgo de Osma, Madrid. I actually could not participate due to health problems.

July 3-7, 2006. *Joint meeting of the Unione Matematica Italiana and the Societé Mathématique de France*, Torino, special session on Geometry and Topology of Groups, one-hour talk on enumeration of knotted graphs and 3-orbifolds.

May 22-27, 2006. Invitation to give a 50-minutes talk on enumeration of knotted graphs and 3-orbifolds in the conference *Knot theory, low-dimensional topology and group geometry*, Marseille. I actually could not participate due to health problems.

April 4, 2006. After an invitation of Ian Agol, one-hour seminar at the University of Chicago on complexity of graphs and 3-orbifolds .

April 1-2, 2006. 1015th sectional meeting of the AMS, Miami, special session on Invariants of low-dimensional manifolds, 30-minutes talk on exceptional Dehn surgery.

March 28, 2006. After an invitation of Weiping Li, one-hour seminar at the Oklahoma State University on complexity of 3-orbifolds and knotted graphs.

January 17-February 7, 2006. After an invitation of Craig Hodgson, three-weeks visit to the University of Melbourne and one-hour seminar at on branched coverings between surfaces.

January 17-22, 2006. Conference *Manifolds at Melbourne*, one-hour talk on exceptional Dehn surgery.

June 2005. Summer school on the *Topology and geometry of* 3*-manifolds*, ICTP, Trieste; 6-hours course on "Introduction to 3-manifold topology."

April 27-30, 2005. *Seventh panellenic conference on geometry*, Samos; 50-minutes talk on enumeration of 3-manifolds.

April 5, 2005. After an invitation of Joan Porti, visit to Barcelona and 75-minutes seminar on hyperbolic 3-manifolds with geodesic boundary.

February 23-26, 2005. *Workshop on* 3*-manifolds and complexity* in Cortona; one-hour seminar on spherical splitting and complexity of 3-orbifolds.

August 24-26, 2004. *The Keldysh conference* in Moscow; 45-minutes seminar on triangulations of hyperbolic 3-manifolds.

June 1-4, 2004. *Colloque tresses et topologie en petite dimension* in Grenoble; 45-minutes seminar on hyperbolic 3-manifolds with geodesic boundary.

May 6-9, 2004. *INTAS workshop on* 3*-manifolds* in Ebernburg; one-hour seminar on Gadgil's approach to the Andrews-Curtis conjecture; one-hour seminar on the complexity of 3-orbifolds.

May 5, 2004. After an invitation of Wolfgang Metzler, visit to Frankfurt and one-hour seminar on triangulations of 3-manifolds.

September 17-20, 2003. *Colloque Tresses* in Dijon; 30-minutes seminar on exceptional Dehn surgery along knots in handlebodies.

May 2003. Series of 5 lectures of two hours each at the University of Paris VII on various aspects of hyperbolic geometry, enumeration of 3-manifolds, and exceptional Dehn filling.

May 23, 2003. One-hour seminar at the University of South Brittany on the enumeration of hyperbolic 3-manifolds with boundary (invitation by Christian Blanchet).

April 18, 2003. One-hour talk on exceptional Dehn filling of hyperbolic 3-manifolds, within the Special semester *Aspects of real and analytic geometry* at the Centro De Giorgi, Pisa.

September 9-20, 2002. Conference *Advanced course on geometric 3-manifolds* (Bellaterra, Barcelona); 50-minutes talk on complexity of hyperbolic 3-manifolds.

June 6-11, 2001. Conference *Tresses* (Luminy, Marseille); 30-minutes talk on hyperbolic 3-manifolds with geodesic boundary.

April 7-9, 2001. Conference *16th British Topology Meeting* (Edinburgh); 40-minutes talk on the complexity of 3-manifolds.

February 15-20, 2001. After an invitation by Wolfgang Metzler, visit to the University of Frankfurt and delivery of two one-hour talks, respectively on the rotation number and Reidemeister-Turaev torsion.

January 29, 2001. After an invitation by Claudio Arezzo, two-hours lecture at the University of Parma on knot diagrams and the rotation number.

November 3-5, 2000. Special session *The topology of 3-manifolds* at the 959th Sectional Meeting of the American Mathematical Society (Columbia University, New York City); 20-minutes talk on the complexity of 3-manifolds.

July 2-7, 2000. Conference *Marcel Grossman IX* (Roma), one-hour introductory lecture to hyperbolic geometry and 30-minutes talk on ideal triangulations.

June 28-30, 2000. After an invitation of Theodor Bröcker, visit to the University of Regensburg; two lectures of two hours each on the complexity of 3-manifolds, and one-hour talk on knot diagrams on spines and the rotation number.

September 13-18, 1999. Conference *XVI meeting of the Italian Mathematical Union* (Napoli); 30-minutes talk on degenerate ideal triangulations.

March 16-27, 1998. After an invitation by Michel Boileau, visit to the Université Paul Sabatier (Toulouse) and delivery of three lectures of two hours each, respectively on contact structures, ideal triangulations, and confoliations

September 26-28, 1997. Special session *Non-Euclidean and space-time geometries* at the 924th Sectional Meeting of the American Mathematical Society (Montreal); 30-minutes talk on partially flat ideal triangulations of cusped hyperbolic 3-manifolds.

September 7-13, 1997. Conference *Niedrigdimensionale Topologie* (Oberwolfach); one-hour talk on the construction of tight contact structures via branched spines.

September 3-6, 1997. Conference *New Italian contributions to Differential Geometry* (Bari); 50-minutes talk on tight contact structures and branched spines.

February 1997. Visit to the MSRI in Berkeley; conference *Computational and algorithmic aspects of 3-dimensional topology*; one-hour talk on branched spines.

June 3-7, 1996. Conference *Differential geometry and topology* (Palermo); 45-minutes talk on branched spines.

April 19-22, 1996. Conference *V Porto Meeting on Knots and Physics* (Porto); 50-minutes talk on branched spines.

April 1995. Two-weeks visit to the International School of Advances Studies in Trieste, and teaching of a 12-hours graduate course on *Graphic presentations of 3-manifolds, Temperley-Lieb algebra, and quantum invariants.*

July 18-22, 1994. Conference *Knots and 3-manifolds* (Luminy, Marseille); 30-minutes talk on bracket invariants of 3-manifolds with boundary.

March 1994. One-hour seminar on graphic presentations of 3-manifolds with boundary (Milano, invitation by Maria Dedò).

March 1994. Two 45-minutes seminars on hyperbolization of link complements and Poincaré's polyhedron theorem (Trieste, invitation by Bruno Zimmermann).

January 1994. Two-weeks visit to the ETH in Zürich after an invitation by Bruno Colbois. Six-hours course on *Thurston's hyperbolic Dehn surgery theorem*, and one-hour seminar on Poincaré's polyhedron theorem.

September 1993. Two one-hour seminars on the hyperbolization of link complements (Parma Physics Department, invitation by Enrico Onofri).

April 1990. Two one-hour seminars on the Jorgensen-Thurston theory of hyperbolic 3-manifolds (Parma, invitation by Giuliana Gigante).

6 Conferences attended

June 28-July 2, 2004. *Fourth European congress of mathematics* (Stockholm).

June 13-16, 2004. Conference *Topology and singularities* (Dijon).

April 1-5, 2003. *Fundamental groups of hyperbolic manifolds* (Les Diablerets, CH).

July 6-12, 2002. *Janos Bolyai conference on hyperbolic geometry* (Budapest).

September 10-15, 2001. *Kleinian groups and hyperbolic manifolds* (Warwick).

June 5-8, 2000. *Journées toulousaines autour des tresses et des nœuds* (Toulouse).

June 21-24, 1999. *Topologie en petite et grande dimension* (Orsay).

June 22-26, 1998. *Low-dimensional topology – The Kirbyfest* (Berkeley).

November 13-15, 1996. *Geometric group theory and low-dimensional topology* (Bochum).

March 4-5, 1996. *Polyhedral approaches to 3-dimensional manifolds* (Paris, IHP).

July 1995. Summer School *Geometry and Physics* (Odense).

July 25-29, 1994. *Geometric methods in low-dimensional topology* (Lyon).

May 19-20, 1994. *Differential Geometry and Complex Analysis* (Parma).

June 20-24, 1990. *International Conference on Topology and its Applications* (Dubrovnik).

7 Administrative and organizational activities

May 20-25, 2007. Joint organizer with Riccardo Benedetti, Roberto Frigerio, Giovanni Gaiffi, and Mario Salvetti of the international meeting *Braids and their Ramifications* in Cortona. The schedule includes talks by about 30 renowned experts in the algebraic and topological branches into which the theory of braids has evolved. Details are available from the the following web page:
`http://www.dm.unipi.it/~gaiffi/braidscortona`

January 2007 – December 2008. Scientific and administrative supervisor of a Marie Curie activity entitled "Complexity and quantum invariants of 3-manifolds and knots." The activity includes a fellowship granted to Ekaterina Pervova at the Department of Applied of Mathematics of the University of Pisa, and it was funded by the EU with a total sum of Eur. 117 300.

January 2007 – December 2008. Scientific and administrative coordinator of a research project entitled "Combinatorial methods based on triangulations in low-dimensional geometric topology and geometric group theory" within the framework of the joint Italian CNR and French CNRS activities. The project, which includes teams from Pisa and from Strasbourg, was approved and granted financial support up to a maximum of Eur. 36 000.

August 22-27, 2005 Member of the scientific committee of the *Workshop on geometry and topology of 3-manifolds* held in Novosibirsk.

April 2005-December 2006. Representative of the Department of Applied Mathematics in the steering board of the Doctorate School in Mathematics of the University of Pisa.

2006. Referee for papers submitted to Geometry and Topology, Geometriae Dedicata, Annali della Scuola Normale Superiore di Pisa.

June 6-24, 2005. Joint organizer with Michel Boileau and Bruno Zimmermann of an activity entitled *Geometry and topology of* 3*-manifolds* at the International Center for Theoretical Physics (ICTP) in Trieste. The activity included two weeks of summer school, particularly addressed to pupils coming from developing countries, and a week-long conference with over 130 participants and 32 talks, given by several of the most prominent researchers in the subject. The ICTP has financed this activity with a budget of Eur. 78 000. The web page of the activity is available from `http://cdsagenda5.ictp.trieste.it/full_display.php?smr=0&ida=a04195`

April-June 2005. Member of a committee of the Department of Applied Mathematics for a global reorganization and revision of the contents of the mathematics courses taught at the Faculty of Engineering of the University of Pisa.

April 26, 2005. External referee and member of the evaluation committee for the doctorate thesis of Guillaume Théret, Université Louis Pasteur, Strasbourg

February 23-26, 2005. Organizer of an INTAS workshop in Cortona on the topology and complexity of 3-manifolds. The activity involved about 30 mathematicians from many different countries and included 14 one-hour seminars over 5 days, together with many informal sessions of joint research.

2005. Referee for papers submitted to Commentarii Mathematici Helvetici, Acta Applicandae Mathematicae, and Rendiconti del Circolo Matematico di Palermo

September 2004. Member of the evaluation committee for admission to the Scuola Normale Superiore di Pisa.

March 2004 – February 2007. International coordinator of an INTAS project entitled "Complexity, algorithms, and computer methods in geometric topology." This project was funded by INTAS with Eur. 110 000 and includes scientific teams from Pisa, Frankfurt, Novisibirsk, Chelyabinsk.

February 2004 – July 2005. Scientific and administrative supervisor of a Marie Curie activity entitled " Use of combinatorial methods in various approaches to three-dimensional topology." The activity includes a fellowship granted to Gwénaël Massuyeau at the Department of Applied of Mathematics of the University of Pisa, and it was funded by the EU with a total sum of Eur. 103 000.

1994 – today. Reviewer of papers for Mathematical Reviews (about 40 papers reviewed so far).

1999 – today. Master of the web page of the research group in topology and geometry of the Department of Mathematics in Pisa, accessible from the following address: `http://www.dm.unipi.it/~geom/`.

2004. Referee for papers submitted to Advances in Geometry, Acta Applicandae Mathematicae, and Edizioni dell'Unione Matematica Italiana.

2002/03 – 2003/04. Elected as a representative of full professors in the permanent committee of the University of Pisa for the assignment of research funds in Mathematics and Computer Science.

November 2003. Member of the evaluation committee for admission to the Doctorate School in Mathematics, University of Pisa.

2003. Referee for papers submitted to the Pacific Journal of Mathematics, the Bulletin of the Mathematical Society of Mexico, Advances in Geometry, the International Journal of Mathematics and Mathematical Sciences, and Experimental Mathematics.

June 19-22, 2002. Joint organizer with Riccardo Benedetti and Mario Salvetti of the international meeting *Braids in Cortona*. The schedule included 4 mini-courses and 20 talks, and about 90 participants attended the conference. Details are available from the home page I have set up for this event at the following address:
`http://www.dm.unipi.it/~geom/meetings_2002/BraCo/BraCo_index.html`

June 11-16, 2002. Joint organizer with Riccardo Benedetti, Jeff Weeks, and Dale Rolfsen of a special session on *The topology of* 3*-manifolds* within the First Joint Meeting of the American Mathematical Society and the Unione Matematica Italiana, in Pisa. A follow-up informal session on June 17 under the same title was also organized. Twenty talks were delivered and about 50 more participants attended the event. Details are available from the home page I have set up for this meeting at the following address:
`http://www.dm.unipi.it/~geom/meetings_2002/AMSUMI/AMSUMI_index.html`

2002. Referee for papers submitted to Geometriæ Dedicata and Mathematisches Nachrichten.

January 2002. External referee and member of the evaluation committee of the PhD dissertation in Mathematics of Guzman Tierno at the University of Rome I.

2001. Referee for papers submitted to Experimental Mathematics, Rendiconti dell'Accademia Nazionale dei Lincei and Atti dell'Accademia dei XL.

1999/2000. Joint organizer with Riccardo Benedetti e Paolo Lisca of a special activity on *Geometric and Symplectic Topology* in Pisa, with one- and two-week visits by A. Stipsicz, D. Kotschick, F. Laudenbach, D. Auroux, L. Kauffman, M. Hutchings, J.-C. Sikorav, R. Piergallini, J. Weeks, B. Zimmermann, M. Reni, S. Vidussi, L. Funar.

1996/97 – 1998/99. Organizer of the visits to Pisa of É. Giroux, J. Weeks, J. Porti, M. Boileau, S. Matveev, D. Repovs, Y. Kawahigashi, I. Murakami, A. Mednykh.

1996/1997 and 1997/1998. Member of the governing board of the Department of Mathematics, University of Pisa.

Spring 1995. Member of the organizing committee of an intensive semester on *Low dimensional topology* in Pisa, with talks and courses by M. Boileau, J.-P. Otal, A. Marin, É. Giroux, D. B. A. Epstein, F. Paulin, V. Turaev, G. Masbaum.

1994/95 – 1997/98. Elected as a representative of research assistants in the permanent committee of the University of Pisa for the assignment of research funds in Mathematics and Computer Science.

1987/88 and 1988/89. Elected as a representative of students in the executive council of the Scuola Normale Superiore.

8 General information

Computing skills
I write my papers using LaTeX.

While working on Thurston's solitaire tilings of the plane [7] I have used Mathematica and written some code to do the relevant computations and draw the pictures of the tilings.

I am running my own web page and that of the geometry and topology research group in Pisa, using the HTML language.

I have carried out part of the hyperbolic graph census [45] using Haskell, a purely functional programming language that I have come to master rather deeply.

Language skills
I speak English and French fluently and I have given many talks and lectures in these languages. I understand Spanish and I speak the very basics of it.

Personal information
Born in Parma, Italy, on November 3, 1966. Married, three children.

9 Scientific activity: completed works

1988-1990: holomorphisms of domains in complex Banach spaces My research activity started under the supervision of Edoardo Vesentini, with an internal seminar at the Scuola Normale and the degree dissertation. I obtained generalizations to various situations of classical non-homogeneity theorems valid for domains in complex two-dimensional space. My results on this topic are collected in three short papers [1, 2, 3].

1990-1992: hyperbolic geometry At the beginning of my PhD studies I switched from complex analysis to the somewhat different subject of hyperbolic geometry. For quite some time I mostly concentrated on the joint writing with Riccardo Benedetti of a graduate textbook on this topic, later published by Springer-Verlag [4]. None of the results our book contains is strictly speaking new, but we have often reworked and simplified the known proofs, sometimes filling gaps and making the arguments more easily accessible. The response to our book has been very favourable worldwide, and the book was reprinted in 1996 and 2003.

1991: hyperbolicity equations Starting from the book written with Benedetti, my mathematical interests have been focused on low-dimensional topology and geometry, and particularly on the theory three-dimensional manifolds, where hyperbolic geometry plays a central rôle. My first original contribution [5] to this field illustrates a very flexible method to find a (topological) ideal triangulation of a link complement in the 3-sphere, and hence to investigate hyperbolicity of the link. I mention here that years later I have extended the method to complete generality [10].

1992: Poincaré's polyhedron theorem As a PhD student I have spent the academic year 1991/92 in Warwick, working with David Epstein on Poincaré's polyhedron theorem, which gives a powerful method for constructing hyperbolic, Euclidean, and elliptic manifolds and orbifolds. We have given a unified proof for all dimensions and geometries, particularly focused on the algorithmic aspects. Our result is published in [6].

1992: tessellations of the plane In Warwick I also worked on a method suggested by Thurston for constructing self-similar tessellations of the plane. This method involves notions coming from theoretical computer science (the theory of finite state automata) and from number theory (the theory of Pisot numbers) and leads to tessellations having tiles of fractal shape. I systematized the ideas of Thurston and produced many examples, publishing my results in [7].

1993-1994: standard spines and quantum invariants After returning from Warwick I resumed my collaboration with Benedetti. We devoted our attention to the theory of standard spines, with the idea of funding a new combinatorial approach to 3-manifolds. Our hope was to apply the combinatorial framework to the investigation of the various topological and geometric theories of 3-manifolds that were spreading at that time, in particular the theory of quantum invariants. As a first step in this project we have introduced a convenient graphic encoding for standard spines and for the moves of the Matveev-Piergallini calculus on spines. This encoding has suggested us the (formal) definition of a class of invariants, called bracket invariants, that contains all the Turaev-Viro invariants. Our results were published in [8]. They also appear in my PhD thesis [9], together with the result already mentioned on ideal triangulations of link complements, published separately in [10]. As a first application of our combinatorial encoding of 3-manifolds we have proved an extension to the case of non-empty boundary of the Turaev-Walker theorem, according to which each Turaev-Viro invariant is just (up to normalization) the squared module of the corresponding Reshetikhin-Turaev-Witten invariant. This result was published in [11].

1994-1995: branched standard spines The next applications of the combinatorial approach of Benedetti and me to spines and 3-manifolds, after quantum invariants, was obtained, starting from 1994, by investigating the relation with the notion of branched surface. It turned out that when a polyhedron is at the same time a branched surface and a standard spine of a 3-manifold, there is a naturally induced geometric structure on the manifold. The rich theory arising from this observation was illustrated in a monograph [12], and in the proceedings of a conference [13]. We have been able to provide combinatorial presentations, according to the objects/moves scheme popular in dimension three, of the following topological categories: combed manifolds, framed manifolds, and spin manifolds. Some remarkable connections of our approach with the theory of contact structures and foliations were also pointed out in the monograph.

1996: contact structures After the publication of the monograph [12], Benedetti and I have concentrated on the relations between our results and tight contact structures, at that time one of the most mysterious and challenging topics in low-dimensional topology. We have shown that under certain circumstances a branched spine can "carry" a tight structure, and we have proved notable uniqueness results, coming from the intrinsically global nature of the topological information provided by both a tight contact structure and a standard spine. Our results appeared in [14].

1996: topology textbook In 1996 I wrote with Chiara de Fabritiis an advanced undergraduate textbook [15] of problems (with solutions) in topology and geometry. We have included short accounts of the theoretical results needed to face the problems, and the detailed solution of 200 problems. The following topics are covered in the book: general topology, homotopy and homology theory, algebraic curves, complex analysis, differential geometry of curves and surfaces, classical groups.

1996-1997: confoliations At the end of 1996, jointly with Riccardo Benedetti and Paolo Lisca, I organized a reading seminar in Pisa on a paper (then a preprint) of Eliashberg and Thurston on certain relations between contact structures and foliations, the theories of which were unified under the new concept of *confoliation*. The main result of their paper is quite astonishing, but many details of the proof were not completely clear. Thanks to our joint work in the seminar and to several conversations I had with Thurston, I was eventually able to write down a self-contained and detailed proof, published in [16]. The main point I added to the argument is a technical result on the holonomy of codimension-1 foliations, valid in any dimension, published separately in [17].

1997: flat hyperbolic triangulations During 1997 I worked with Jeff Weeks on geodesic ideal triangulations of cusped hyperbolic three-manifolds. Existence of these triangulations is commonly believed and experimentally true, but not proved in general. Existence is however known if one allows the triangulation to include some flat tetrahedra. Having this fact in mind as a motivation, we have investigated the geometric properties of partially flat solutions of hyperbolicity equations. We have obtained the surprising result that consistency equations alone are not sufficient, in the partially flat case, to ensure existence of an (incomplete) hyperbolic structure, because topology can degenerate. However, as one could reasonably expect, if also the completeness equations are satisfied then the solution does provide a hyperbolic structure. Our paper appeared in [18].

1998: hyperbolic Dehn filling via inverted triangulations In the Spring of 1998 I started collaborating with Joan Porti on a proof of Thurston's hyperbolic Dehn filling theorem, based on the technology of ideal triangulations only, and without the assumption of existence of a genuine triangulation with all positive-volume tetrahedra. We have employed a particular partially flat triangulation obtained as a subdivision of the canonical Epstein-Penner cellularization. Using also the techniques developed by Weeks and me, we have been able to provide a completely self-contained and comparatively elementary proof of the filling theorem. In particular, we have given a new proof of the fact that the hyperbolic filling coefficients constitute an open set, without assuming smoothness of the complete point in the variety of deformations. Smoothness is known within the approach through representations of the fundamental group, but the existing proofs of smoothness in the context of triangulations appear to be incomplete. Our paper was published as [19].

1998-2000: Reidemeister-Turaev torsion Starting from 1998 Benedetti and I have been working on a refinement of an invariant (due to Turaev) that, in a suitable sense, lifts the Reidemeister torsion. Turaev's torsion was originally defined for "Euler structures," that is for non-singular vector fields viewed up to homotopy and modifications supported within balls, on manifolds that are closed or bounded by tori, with the extra requirement that the field should never be tangent to the boundary. After several partial results and improvements, which caused two revisions of the first August 1998 preprint, the last of which dated February 2000, we have been able to extend the definition of torsion, and the fundamental H_1-equivariance property, to Euler structures having arbitrary tangency lines to the boundary. The key technical tools of our refinement are the extension of the classical Poincaré-Hopf theorem to fields with boundary tangency and the definition of the combinatorial counterpart of a smooth Euler structure. Two more ingredients are however crucial, namely the proof that torsion can be computed directly from a branched spine, and the remark that our generalized torsions apply in particular to pairs field-link such that the link is transversal to the field. In this context we have also shown that, for a knot in a homology sphere, our torsion contains a lifting of the classical Alexander invariant. Our paper appeared in [20].

1999: fatherhood probability During 1999 I cooperated with Nicola Cucurachi, a research assistant in Legal Medicine at the University of Parma. We have jointly authored a paper about paternity tests through genetic analysis. I have been often called by Cucurachi for the solution of probability problems arising in this type of tests, and I have observed that medical papers often contain errors, sometimes gross ones. Our paper tries to put straight some of the points most often wrongly addressed in the literature. The mathematical content of the paper is of course very elementary, but the effort of mutual understanding between languages and cultural environments as far from each other as Mathematics and Medicine was quite stimulating for me. Our paper appeared in [21].

1999: degenerate hyperbolic triangulations At the end of 1999 I collected in a new article the main points of the papers with Weeks and Porti. I concentrated on the questions remained open, addressing in particular the (well-known and hard) problem of a general understanding of the topological and geometric meaning of a degenerate (partially inverted) solution of hyperbolicity equations. This article contains some new results and it was presented in a seminar I gave at a conference. It appeared in [22].

1999-2000: Euler structures with boundary and framed knots Between the end of 1999 and the beginning of 2000, stimulated by our investigations on Reidemeister torsion for Euler structures, Benedetti and I have succeeded in extending our calculus for combed manifolds from the closed to the bounded case. We have also provided a combinatorial encoding for pairs (non-singular field)–(transversal link) up to homotopy of the field and transversal isotopy of the link. These results, together with the refinement of Turaev's calculus for framed knots represented by diagrams on branched spines, have allowed us to provide an extension of the notion of 'winding number' of a knot diagram. A remarkable fact is that we can define this number not only when the ambient manifold is a homology sphere, but also for most knots having hyperbolic complement. This paper appeared in [23].

1999-2000: Feynman diagrams and topology In the Summer of 1999 I started working with Roberto De Pietri, a physicist in Parma, on the topological interpretation of the Feynman diagrams arising from the expansion of a certain partition function, proposed by De Pietri himself with Rovelli, Krasnov, and Freidel. From the mathematical viewpoint the problem reduces to the issue of checking whether certain simplicial 4-complexes are 4-manifolds with boundary. In general this issue contains the (very hard) problem of recognizing the 3-sphere, but the 4-complexes arising in the physical setting were actually simple enough that we could give a definite and algorithmic answer to the problem. Our work appeared in [24].

1999-2000: enumeration of 3-manifolds In the Fall of 1999 I started collaborating with my graduate student Bruno Martelli on a project of enumeration of 3-manifolds of small complexity. I am referring here to Matveev's complexity, defined by means of standard spines and equal, for irreducible manifolds, to the minimal number of tetrahedra in a triangulation. Martelli carried out extensive computer experiments, and the analysis of the data he collected suggested us a very powerful structure theorem for 3-manifolds, later proved in full generality in the framework of a new theory of complexity. Namely, we have shown that there exist certain manifolds with (suitably marked) toric boundary, that play the rôle of elementary "bricks," in the sense that all 3-manifolds are obtained by (suitably) assembling these manifolds. What is actually most remarkable (and very important from the computational viewpoint) is that, in any given complexity, the number of bricks is dramatically smaller than the number of all manifolds. For instance, there are 1155 manifolds in complexity 9, and (essentially) 7 bricks only are already sufficient to generate all of them. Another notable feature of our decomposition is that all the bricks we have found so far are geometrically atoroidal. This implies that, for manifolds obtained from these bricks only, our decomposition is actually a refinement of the celebrated Jaco-Shalen-Johannson decomposition, that provides the topological starting point of Thurston's geometrization conjecture. Our theoretical and experimental results appeared in [25].

2000-2001: non-orientable 3-manifolds and bricks Between the end of 2000 and the beginning of 2001 I have continued working with Bruno Martelli on the complexity of 3-manifolds, proving the non-orientable analogue of the decomposition theorem into bricks mentioned above. Extending a result from the orientable to the general case is sometimes straight-forward or even tedious, but this was definitely not true in our case. We have observed in particular that accepting Klein bottles on the boundary entails a variety of new and very interesting topological phenomena, that have no orientable counterpart. In particular, finiteness of the number of irreducible manifolds in any given complexity is only true in a more refined sense than in the orientable context. This has forced us to give a somewhat more accurate definition of the notion of brick, and to develop new techniques for the proof of the decomposition theorem. Our paper was published in [26].

2000: combinatorial Reidemeister torsion In the Fall of 2000 Benedetti and I have been working with our students Gennaro Amendola and Francesco Costantino on a new purely combinatorial approach to the theory of the Reidemeister-Turaev torsion, using our calculi for manifolds and knots. This approach has the advantage of allowing to define torsion in situations more general than the "classical" ones already considered in [20]. It also has the drawback that the topological information carried by torsion in the classical cases —but not in the closed case— is weaker. Our paper appeared as an invited one in a special volume [27].

2001: hyperbolicity equations with boundary Starting from the Spring of 2001 I have been working with my student Roberto Frigerio on partially truncated ideal triangulations of hyperbolic 3-manifolds with geodesic boundary. The theory of these manifolds and their triangulations, which has at its core Kojima's result on the existence of a canonical decomposition, is particularly rich when both the manifold and its boundary are non-compact. We have extended the theory of moduli and hyperbolicity equations to this setting, discovering in particular a rather surprising interpretation to give to the notion of ideal triangulation in the case of non-compact boundary. We have also generalized and adapted a procedure (originally described by Weeks in the cusped case) for the transformation of an arbitrary decomposition into the canonical one, using the so-called tilt formula already established by Ushijima. Our results were published in [29].

2001-2003: exceptional Dehn surgery on the 3-chain link The decomposition theory developed by Martelli and me involves four bricks having complexity 9. Two of them have as underlying space a well-known manifold, called the “magic” one by Cameron Gordon, which is the complement in the 3-sphere of a 3-component link whose components are individually unknotted. The other two bricks of complexity 9 are obtained by hyperbolic Dehn surgeries along this link. Using the technology of spines and skeleta we have then been able to completely classify the (infinitely many) non-hyperbolic surgeries along this link (not just the slopes, also the resulting manifolds). Analyzing the nature of these surgeries we have been able to show that most known upper bounds between different types of exceptional fillings on hyperbolic manifolds are realized by the “magic” one. The first 2002 version of our preprint underwent a substantial revision in 2003 and was eventually published in [30].

2002-2003: complexity of geometric 3-manifolds Again with Martelli I have been working, starting from the Fall of 2002, on upper bounds on the complexity of general 3-manifolds obtained by our decomposition theorem into bricks. The idea is that the bricks we already have, even if certainly not sufficient to construct all hyperbolic manifolds, already allow to construct huge classes, including all Seifert fibered spaces and torus bundles, and therefore provide upper bounds for the complexity of these manifolds. With a careful search for the “best” realization via the bricks known so far, we have actually found a precise formula for a certain “approximated complexity” for Seifert spaces and torus bundles [31] . We actually conjecture this formula to give the exact value of complexity except for some very special atoroidal Seifert manifolds. This conjecture extends and unifies previous ones of Matveev and Anisov.

2002-2004: enumeration of hyperbolic manifolds with boundary Building on one side on the theoretical results [29] on hyperbolicity equations and the tilt formula, and on the other side on the computer programs already used in the closed case [25], I have developed with Frigerio and Martelli a software which has enabled us to list hyperbolic manifolds with geodesic boundary up to complexity 4. We have confined ourselves to manifolds having non-empty *compact* boundary, thus avoiding annular cusps, but we have allowed toric cusps. After reproducing the known census in complexity 2, which comprises 8 compact manifolds, we have found 151 in complexity 3 (one of which has a toric cusp) and 5033 in complexity 4 (30 having one toric cusp and one having two). For all the manifolds in the census, using the tilt formula and a recent volume formula of Ushijima, we have computed the Kojima canonical decomposition and the volume, discovering many remarkable combinatorial and numerical data. Our results were published [33] by a specialized journal. A systematic approach to the enumeration issues, which includes the case of manifolds with non-compact boundary, was provided in [38].

2002: exact complexity computations The experimental data found in [33] suggested Frigerio, Martelli and me to consider the special class $\mathcal{M}_n$ of 3-manifolds with boundary of genus n which can be ideally triangulated with n tetrahedra. We have then shown that the elements of $\mathcal{M}_n$ have complexity n and Heegaard genus $n+1$. Moreover, we have proved that they are all hyperbolic with geodesic boundary, that they all have the same volume, and that their ideal triangulation with n tetrahedra actually is the Kojima canonical decomposition. This last property implies that different ideal triangulations give different elements of $\mathcal{M}_n$. Using this fact and some combinatorics of standard spines, we have shown that $\#(\mathcal{M}_n)$ grows exponentially with n, so indeed this class $\mathcal{M}_n$ is rather large. Our paper [32] describes all these results.

2003: exceptional fillings of large manifolds In another paper [34] completed in the Spring of 2003 and accepted shortly after submission, Frigerio, Martelli, and I have pursued our analysis of hyperbolic 3-manifolds, introducing the class $\mathcal{M}_{g,k}$ of 3-manifolds M such that ∂M consists of one surface of genus g and k tori, and M can be ideally triangulated with $g+k$ tetrahedra. We have then extended to $\mathcal{M}_{g,k}$ the results proved in [32] for $\mathcal{M}_n$, namely that $M \in \mathcal{M}_{g,k}$ has complexity $g+k$ and Heegaard genus $g+1$, that M is hyperbolic with geodesic boundary and k toric cusps, that the volume of M is a function of g and k only, and that the triangulation of M with $g+k$ tetrahedra is Kojima's canonical decomposition, so it is unique. We have then shown that $\#(\mathcal{M}_{g,k})$ grows factorially in g when k is fixed (thus improving the exponential growth estimate established in [32]). Moreover we have completely analyzed the exceptional Dehn fillings of the cusps of the elements of $\mathcal{M}_{g,k}$. This has enabled us, among other things, to prove a conjecture of Wu on the largest possible distance between an annular and a boundary-reducible Dehn filling of a "large" hyperbolic 3-manifold, where "large" means that the homology is not that of a link complement.

2003-2004: geometric investigation of triangulations In a paper [35] completed in February 2004 with Costantino, Frigerio, and Martelli, we have analyzed the set of combinatorial triangulations in geometric terms. More precisely, we have shown that one can canonically associate to a triangulation T a relative hyperbolic handlebody $N(T)$ ($i.e$ a pair (H,Γ), where H is a handlebody, Γ is a set of curves on ∂H, and $H \setminus \Gamma$ is hyperbolic with totally geodesic boundary), and a cusped hyperbolic manifold $D(T)$, which actually is a link complement in a connected sum of copies of $S^1 \times S^2$. Moreover, using the canonical decompositions of Epstein-Penner and Kojima, we have proved that the geometries of $N(T)$ and $D(T)$ both determine T. As applications of our constructions we have shown that:

- a triangulation (which may have multiple and self-adjacencies, finite and/or ideal vertices) is determined up to isotopy by its 1-skeleton;
- a triangulation with all edges having valence at least 6 defines a hyperbolic structure on which all the edges of the triangulation can be isotoped to geodesics;
- there exists a constant $c > 0$ such that any finite group G is the full isometry group of a hyperbolic manifold M with $\mathrm{vol}(M) < c \cdot |G|^9$.

2004: complexity of 3-orbifolds In the Spring of 2004 I started working on a generalization to 3-orbifolds of Matveev's complexity theory for 3-manifolds, based on special spines. After giving a rather natural definition of complexity, I succeeded in proving the orbifold analogues of several of the main results valid for manifolds [37]. In particular, I showed that for any n there is only a finite number of irreducible orbifolds up to complexity n. I also proved that, with some well-understood exceptions, the minimal spines of these orbifolds are special, whence dual to triangulations, which shows that the definition is very natural. These results prompt for computer investigation of orbifolds, which I am planning to pursue in the future. Concerning the additivity of complexity under connected sum, which holds for manifolds, I was able to prove it under ordinary and cyclic connected sums. I also proved that additivity cannot hold in a strict sense for vertex connected sum, but I established 2-sided linear estimates on the behaviour of complexity under this operation.

2004: spherical splitting of 3-orbifolds While working on additivity of complexity, I realized that no satisfactory account of the orbifold analogue of the Haken-Kneser-Milnor theorem existed in the literature. This theorem asserts that every 3-manifold splits as a connected sum of prime ones, and the summands are unique provided none of them is the 3-sphere. The orbifold analogue was believed to be true for many years, but it was neither clearly stated nor correctly proved. And it turned out that neither uniqueness nor existence of the splitting are true if one takes the straight-forward generalization, which excludes *tout-court* the spherical 3-orbifolds as possible summands, because, in a suitable sense, the operations of connected sum of different types do not commute with each other. Even after overcoming this problem of non-commutativity, existence itself of the splitting remains not clear when the orbifold contains non-separating spherical 2-orbifolds. Moreover existence cannot be proved using maximal essential systems of spherical 2-orbifolds, as one does in the manifold case. Restricting to orbifolds which contain no bad 2-suborbifolds and in which every spherical 2-orbifold is separating, I have then given a precise statement of the splitting theorem [36], also providing a detailed proof of existence and uniqueness, which uses a suitable adaptation of the theory of normal surfaces to orbifolds, together with the idea of cutting along spherical 2-orbifolds in increasing order of complication.

2004: complexity of groups Upper estimates on the complexity of a given 3-manifold are experimentally very easy to find, and they typically lead to the exact value. On the other hand, lower estimates are much harder to obtain. Besides those connected to geometry, some bounds can be obtained by considering the fundamental group of the manifold and a suitable notion of complexity for this group. These notions were investigated in the preprint [39] with Ekaterina Pervova, where it was shown that for all Abelian groups, and for all the fundamental groups of the spherical 3-manifolds, the complexity is asymptotically equal (up to linear maps) to the order of the torsion of the Abelianization. As an application, an asymptotically exact estimate of the complexity was provided for certain infinite families of lens spaces and more general Seifert spaces.

2004-2006: Existence of branched coverings Starting from the Fall of 2004 I started collaborating with Pervova on a very classical problem in the topology of surfaces, known as the *Hurwitz existence problem*. The question is whether there exist a branched covering between closed surfaces matching some preassigned combinatorial requirements. Some necessary conditions, which reduce to the Riemann-Hurwitz formula at least in the orientable case, have been known for a long time, but these conditions were discovered not to be sufficient when the base surface is the sphere, and the knowledge of effective sufficient conditions has been lacking for over a century. Pervova and I have employed three new tools to attack the problem, namely Grothendieck's dessins d'enfants, the idea of factorizing a covering through two non-trivial ones, and a geometric (graph-theoretic) reinterpretation of the existence problem. Using these tools, in the paper [40] accepted for publication in Algebraic and Geometric Topology, we have found several previously unknown cases where the Riemann-Hurwitz conditions are not sufficient, and we have established the sufficiency of conditions having different nature from those appearing in the literature. More such results were obtained in the preprint [43] of November 2006, now submitted for publication, where a completely different approach, based on recent ideas of Baránski, was exploited. Some of our results were also described in the announcement [41].

2005-2006: Asymptotic complexity estimates for hyperbolic manifolds The value of Matveev's complexity is only known, in the closed case, for a finite number of 3-manifolds, so the issue of computing it for infinite families is a very natural one. In the preprints [42] (joint with Matveev and Vesnin) and [44] (joint with Vesnin) I have attcked the question using tools from hyperbolic geometry. The papers contain asymptotic lower and upper bounds for the complexity of three infinite families of hyperbolic manifolds, namely Fibonacci manifolds, Löbell manifolds, and branched covers of two-bridge links. The two bounds differ by fixed (and not too large) factors, so the estimate can be considered to be asymptotically exact up to linear terms.

10 Scientific activity: work in progress and projects

Enumeration of hyperbolic knotted graphs and 3-orbifolds Hodgson (Melbourne) and I have started in January 2006 the theoretical work underlying a census of "small" hyperbolic 3-orbifolds. As we were discussing, we understood that the notion of orbifold complexity I have introduced is actually not perfectly suited to the task, and we have worked out a variation which applies to 3-valent (knotted) graphs in 3-manifolds, without any decoration of the edges. After completing this theoretical work, in collaboration with our students Heard and Martelli, we have implemented the census, starting with the case of graphs. We have completely classified the hyperbolic ones (in arbitrary closed

3-manifolds, with hyperbolicity meant in the sense of parabolic edge meridians with thrice punctured geodesic boundary spheres) up to complexity 5 (with the restriction that the graph should have no knot component when the complexity is exactly 5), getting many very interesting objects, that we have been able to completely describe together with their many geometric and algebraic invariants coming from the hyperbolic structure. We are in the process of writing the paper describing these results [45]. Hodgson will visit Pisa for a month in 2007 and on that occasion we will proceed with the census of hyperbolic orbifolds, choosing orders for the edges of the graphs and then computing the hyperbolic structure, or proving it does not exist.

Complexity of knots Using my results on spherical splitting and complexity of 3-orbifolds, I am working with Pervova on the corresponding new notion of complexity for knots. We have shown that this new invariant is additive under connected sum, both along and away from the knot, and that there are upper and lower bounds of complexity in terms of the crossing number. We also have conjecturally exact upper bounds for the complexity of any torus knot in a lens space.

Minimal triangulations of hyperbolic manifolds Francaviglia and I are trying to prove that in the cusped hyperbolic case a minimal spine, *i.e.* one that realizes Matveev's classical complexity, is dual to an ideal triangulation that is *geometric* in a suitable sense (maybe partially flat or even partially inverted, but not geometrically degenerate). We have been able to associate to any triangulated hyperbolic manifold another triangulated manifold, that we are trying to prove to be the original one when the original triangulation is minimal.

Geometric structures via triangulations With Papadopoulos I would like to extend to a 3-dimensional context his results on the construction of transversely affine and measured foliations on triangulated surfaces. The idea is to define the appropriate structure on the ideal tetrahedra of a triangulation, and to glue the tetrahedra via affine maps.

Hyperbolic graph complements I have introduced an infinite family of graphs in the 3-sphere which includes Thurston's first example of a hyperbolic manifold with geodesic boundary. I have devised a very general procedure to triangulate a graph complement and I have applied it to the graphs of this family, thus getting a triangulation for each graph complement. After computer experiments, I have convinced myself that all these spaces actually are hyperbolic and that their canonical Kojima decomposition is strictly related to my triangulation. I am now trying to give a theoretical argument to prove this fact for all the infinitely many members of the family.

The 5-component chain-link Within their investigations on the virtual Haken conjecture, Dunfield and Thurston remarked that almost all known closed hyperbolic manifolds are obtained by Dehn surgery along the minimally twisted 5-component chain-link. This link is therefore crucial to hyperbolic geometry (or, at least, for the portion we now know). Martelli and I are planning to use techniques similar to those employed in [30] to completely analyze the exceptional surgeries along this link. We have already found a spine of the link complement which reflects most of the many symmetries of the link, and we are hoping to base on this spine our analysis, whose results may even be easier to describe than those of [30], due to the larger group of symmetries involved.

Behaviour of complexity under finite covering Using some explicit constructions of finite coverings between torus bundles and Seifert fibered spaces, together with the conjectural formulae of Martelli and me [31] for the complexity of such manifolds, I would like to collect some experimental evidence on the way complexity behaves under finite covering. This may open other branches of investigation, particularly on connections between complexity and domination between 3-manifolds. I expect to use the results obtained with Pervova on the existence of branched coverings between surfaces, which translates into the existence of orbifold-coverings between the base 2-orbifolds of Seifert fibrations.

Complexity in low-dimensional topology The aim of the international network I have set up with Wolfgang Metzler (Frankfurt), Sergei Matveev (Chelyabinsk), Alexander Mednykh (Novosibirsk), and their many students and collaborators, is to develop the theory of complexity in several direction, and to discover relations between this theory and other branches of geometry and topology. Some of these new developments, such as those dealing with groups and with orbifolds, have already been found, as mentioned above, and I expect several more specific projects to arise in the forthcoming years.

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