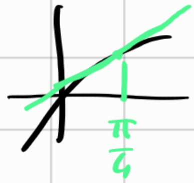
ANALISI MATEMATICA B
LEZIONE 52 - 15. 2. 2023

$x \to 0$ II° compitino 20.2.2021

$$f(x) = \frac{\cos(x\sqrt{2}) - \sqrt{2} \cdot \sin \operatorname{arctg} \cos 2x}{(e^x + e^{-x})^5 - 32 - 80x^2}$$



$$\boxed{(1+x)^5 = 1 + 5x + 10x^2 + o(x^2) \quad x \to 0}$$

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + o(x^4)$$

$$e^{-x} = 1 - x + \frac{x^2}{2} - \frac{x^3}{6} + \frac{x^4}{24} + o(x^4)$$

$$(e^x + e^{-x})^5 = \left(2 + x^2 + \frac{x^4}{12} + o(x^4)\right)^5$$

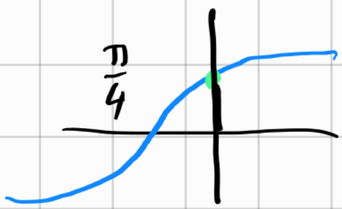
$$= 2^5 \cdot \left(1 + \frac{x^2}{2} + \frac{x^4}{24} + o(x^4)\right)^5$$

$$= 2^5 \cdot \left(1 + 5\left(\frac{x^2}{2} + \frac{x^4}{24} + o(x^4)\right) + 10\left(\frac{x^2}{2} + o(x^2)\right)^2 + o(x^4)\right)$$

$$= 32\left(1 + \frac{5}{2}x^2 + \underbrace{\frac{5}{24}x^4 + 10 \cdot \frac{x^4}{4}} + o(x^4)\right)$$

$$= \underbrace{32 + 80x^2} + \frac{260}{3}x^4 + o(x^4)$$

$\operatorname{arctg} \cos 2x$ ⚠ $\cos 2x \to 1$ per $x \to 0$



Mi interessa il pol. di Taylor di $\operatorname{arctg} x$ per $x \to 1$.

$$\operatorname{arctg} \cos(2x) = \operatorname{arctg}(1 - 2x^2 + o(x^2))$$

$$f(x) = f(x_0) + f'(x_0)\cdot(x-x_0) + \frac{f''(x_0)}{2}(x-x_0)^2 + o((x-x_0)^2)$$

$x_0 = 1$

$f(x) = \operatorname{arctg} x \qquad f(1) = \frac{\pi}{4}$

$f'(x) = \frac{1}{1+x^2} \qquad f'(1) = \frac{1}{2}$

$f''(x) = -\frac{2x}{(1+x^2)^2} \qquad f''(1) = -\frac{2}{4} = -\frac{1}{2}$

$\to \operatorname{arctg} x = \frac{\pi}{4} + \frac{1}{2}(x-1) - \frac{1}{4}(x-1)^2 + o((x-1)^2)$ per $x \to 1$

Forse meglio scrivere così:

$$\operatorname{arctg}(1+x) = \frac{\pi}{4} + \frac{1}{2}x - \frac{1}{4}x^2 + o(x^2) \quad \text{per } x \to 0$$

$$\operatorname{arctg}\cos(2x) = \operatorname{arctg}\left(1 \boxed{-2x^2 + \frac{2}{3}x^4 + o(x^4)}\right)$$

$$= \frac{\pi}{4} + \frac{1}{2}\left(-2x^2 + \frac{2}{3}x^4\right) - \frac{1}{4}\left(-2x^2 + o(x^2)\right)^2 + o(x^4)$$

$$= \frac{\pi}{4} - x^2 + \frac{x^4}{3} - \frac{1}{4}4x^4 + o(x^4)$$

$$= \frac{\pi}{4} - x^2 - \frac{2}{3}x^4 + o(x^4)$$

$$\sin(\operatorname{arctg}\cos(2x)) = \sin\left(\frac{\pi}{4} - x^2 - \frac{2}{3}x^4 + o(x^4)\right)$$

$$\sin\left(\frac{\pi}{4} + \alpha\right) = \sin\frac{\pi}{4}\cos\alpha + \cos\frac{\pi}{4}\sin\alpha$$

$= \ldots$

oppure calcoliamo il pol. di Taylor

$$\sin\left(\frac{\pi}{4} \boxed{+x}\right) = \sin\frac{\pi}{4} + \cos\frac{\pi}{4}\cdot x - \frac{\sin\frac{\pi}{4}}{2}x^2 + \boxed{o(x^2)} \quad \text{per } x \to 0$$

$$\xrightarrow{D} \cos\left(\frac{\pi}{4}+x\right) \xrightarrow{D} -\sin\left(\frac{\pi}{4}+x\right).$$

$$\sin\left(\arctan \cos(2x)\right) = \sin\left(\frac{\pi}{4} \boxed{- x^2 - \frac{2}{3}x^4 + o(x^4)}\right)$$

$$= \sin\frac{\pi}{4} + \cos\frac{\pi}{4}\cdot\left(- x^2 - \frac{2}{3}x^4\right) - \frac{\sin\frac{\pi}{4}}{2}\cdot x^4 + o(x^4)$$

$$= \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}x^2 - \frac{\sqrt{2}}{\cancel{2}}\frac{\cancel{2}}{3}x^4 - \frac{\sqrt{2}}{4}x^4 + o(x^4)$$

$$= \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}x^2 - \frac{7}{12}\sqrt{2}\,x^4 + o(x^4)$$

$$f(x) = \frac{\cos(\sqrt{2}x) - 1 + x^2 + \frac{7}{6}x^4 + o(x^4)}{\frac{260}{3}x^4 + o(x^4)}$$

$$= \frac{\left(\cancel{1} - \cancel{x^2} + \frac{x^4}{6} + o(x^4)\right) \cancel{-1} \cancel{+x^2} + \frac{7}{6}x^4 + o(x^4)}{\frac{260}{3}x^4 + o(x^4)}$$

$$= \frac{\frac{4}{3}x^4 + o(x^4)}{\frac{260}{3}x^4 + o(x^4)} \longrightarrow \frac{4}{260} = \frac{1}{65}$$

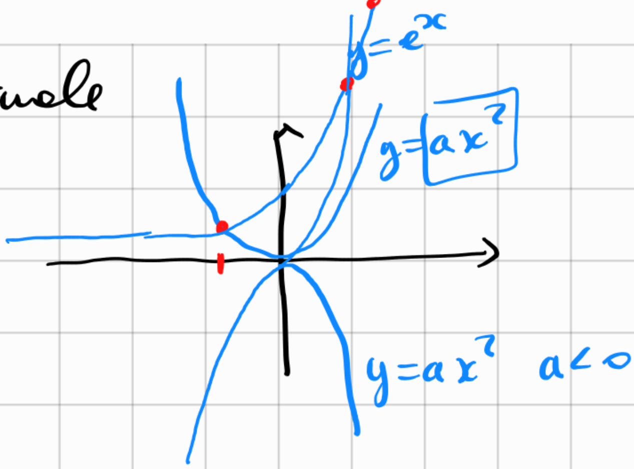
$$e^{2+x} = e^2\cdot e^x$$

$$\ln(7+x) = \ln\left(7\cdot\left(1+\frac{x}{7}\right)\right) = \ln 7 + \ln\left(1+\frac{x}{7}\right)$$

$$(42+x)^\alpha = 42^\alpha\cdot\left(1+\frac{x}{42}\right)^\alpha$$

ES V test settimanale

$$e^x = a \cdot x^2$$



per $x \to +\infty$ $\quad e^x \gg a \cdot x^2$

Voglio sapere quante sol. ha l'eq. $\boxed{e^x = a \cdot x^2}$ al variare del parametro a.

Idea: ricondursi ad uno studio di funzione.

Idea 1. $f_a(x) = e^x - ax^2 = 0$ ← questo è complicato

Idea 2. $\quad \boxed{\dfrac{e^x}{x^2} = a}$ ←

Idea 3

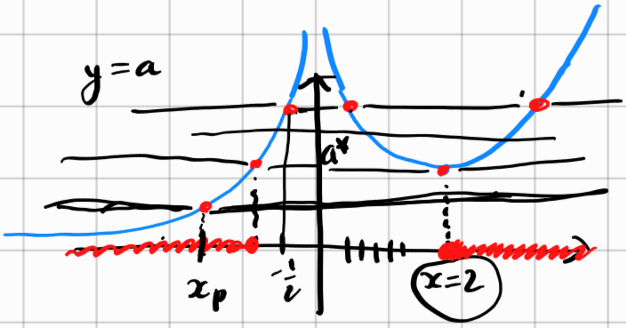
$$x = \ln(a \cdot x^2) \quad \leftarrow$$

$$x = \ln a + 2 \ln|x|$$

$$x - 2\ln|x| = \ln a$$

Seguiamo Idea 2 $\qquad f(x) = \dfrac{e^x}{x^2}$ $\qquad x \neq 0$

$$(-\infty, 0) \cup (0, +\infty)$$



$$\lim_{x \to -\infty} f(x) = 0$$

$$\lim_{x \to 0} f(x) = +\infty$$

$$\lim_{x \to +\infty} f(x) = +\infty$$

$$f'(x) = \frac{e^x \cdot x^2 - e^x \cdot 2x}{x^4} = \frac{e^x(x-2)}{x^3}$$

x:		0		2	
$f'(x)$	$+$	$\nexists$	$-$	0	$+$
$f(x)$	$\nearrow$		$\searrow$	min	$\nearrow$

$a_* = f(2) = \frac{e^2}{4}$ Per $a > \frac{e^2}{4}$ ci sono 3 soluzioni

x_1, x_2, x_3

$x_1 < 0 < x_2 < 2 < x_3$

- perché in $(-\infty, 0)$ f è strett. crescente e assume tutti i valori in $(0, +\infty)$
  se $a > 0$ $\Rightarrow \exists!\, x_1 < 0$ t.c. $f(x_1) = a$

- in $(0, 2]$ f, continua, assume tutti i valori in $[a^*, +\infty)$
  se $a \geq a^*$ essendo strett. decresc. $\forall a \in [a^*, +\infty)$
  $\exists!\, x_2 \in (0, 2]$ t.c. $f(x_2) = a$.

- in $[2, +\infty)$ f continua .... etc.
  ... $\exists!\, x_3 \in [2, +\infty)$ t.c. $f(x_3) = a$
  se $a \geq a^*$

Per $a = \frac{e^2}{4}$ ci sono 2 sol.

$x_1 < 0 < x_2 = 2$

Per $0 < a < \frac{e^2}{4}$ c'è una unica sol. $x_1 < 0$.

Per $a \leq 0$ non ci sono soluzioni

Dimostrare che $x = -\frac{1}{2}$ è sol. dell'eq.
per un valore a tc. ci sono soluzioni più grandi

$$\frac{e^x}{x^2} = \frac{e^{-\frac{1}{2}}}{\frac{1}{4}} \overset{?}{>} a^*$$

$\square$