ON ORTHOGONAL LÉVY MARTINGALES AND MALLIAVIN CALCULUS BASED ON CHAOS

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Topic #5: *Nonstandard Methods in Measure Theory, Stochastic Analysis, Probability and Statistics.*

Based on orthogonal Lévy martingales and the resulting chaos decomposition, we develop Malliavin calculus for a large class of Lévy-processes $(L_t)_{t\in[0,\infty[}$. Without loss of generality the underlying probability space is the *-extension of the set of real sequences (in case of 1-dimensional Lévy processes). The probability measure is the Loeb measure μ_L over the internal product μ of a suitable internal Borel measure μ^1 on $^*\mathbb{R}$. This measure μ^1 is the image measure of the increments of an internal representation $(L_t)_{t\in{}^*\mathbb{N}}$ of $(L_t)_{t\in[0,\infty[}$. In many cases, where polynomials are not integrable or the increments of the processes are unbounded, we use orthogonal polynomials in Borel bijections on the real numbers, which are integrable and bounded. Several examples will be presented, including Brownian motion and the classical Poisson process. One of the examples shows that, in contrast to Brownian motion and Poisson processes, stochastic integration is needed with respect to Lévy martingales, whose increments are orthogonal polynomials of arbitrarily finite degree.

Now the Malliavin derivative is a densely defined closed operator from $L_2(\mu)$ into the Hilbert space of two-parameter processes, depending on time and the orthogonal polynomials. We also introduce the Skorohod integral in this setting and we sketch the proof of a version of the Clark Ocone formula.

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