SOME NONSTANDARD NOTES ON THE FIXED POINT PROPERTY IN THE PLANE

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The proposition below is motivated by what appears to be a promising line of attack on a well-known open question in geometric topology: that every plane continuum (i.e. a compact, connected and co-connected subset of the plane) has the fixed point property. I will outline the proof of the proposition and also discuss its connection with the fixed point problem.

Definition. We will write ∂A for the boundary of a set A, and $\overline{A}$ for the closure of A.
We will write $B(a, r)$ for the open ball about a of radius r.
We will write $\mathcal{C}(a, A)$ for the connected component of A containing a.

Proposition. *Let $E \subset \mathbb{R}^2$ be compact, connected and have a connected complement in the plane. Let f be a continuous function from E to E with no fixed point. Given $\varepsilon > 0$, let U be a bounded connected component of ${}^*\mathbb{R}^2 - {}^*E - B(a, \epsilon)$ for some $a \in {}^*\mathbb{R}^2$, and*

$$Y = \partial(U) \cup \left\{ p : p \in \mathcal{C}(q, \overline{B(a, \delta)}) \text{ for some } q \in \partial(U) \cap B(a, \epsilon) \right\}$$

Then for all $k \in \mathbb{N}$ there does not exist a set of points $a_1, a_2, ...a_k \in {}^*\mathbb{R}^2$ such that each a_i is in Y, and $f(a_1) \approx a_2, f(a_2) \approx a_3, ...f(a_k) \approx a_1$.

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