CANARD SOLUTIONS NEAR A DEGENERATED TURNING POINT

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Topic #7 : *Nonstandard Methods in Differential Equations.*

We are interested in the study of canard solutions of real singularly perturbed differential equations of the form

$$\varepsilon y' = \Phi(x, y, a, \varepsilon) \tag{1}$$

where a is a real parameter, and ε is an infinitesimal real positive number.
By taking the hypothesis that there is a slow curve which has a degenerated turning point (*the slow curve is stable to the left and unstable to the right*), we are searching for the existence of a solution that leave in the ε-galaxy of the so-called *slow curve.*
Following a similar method used in a paper by Benoît, Fruchard, Schäfke and Wallet to study this kind of problem with complex variables and with p parameter, where p is an indication of how degenerated the turning point is, we prove the existence of canard solutions of (1) by applying a fixed point theorem to an iterating operator originated from (1) which is a ($£\varepsilon^{1/(p+1)}$)-Lipschitz function. This result allow us to give a formal method to calculate canard solution of (1).
Furthermore, this allow us to conjecture that the canard solution has an η-asymptotic expansion (with $\eta = \varepsilon^{1/(p+1)}$) :

$$u^* \approx \sum_k u_k \eta^k$$

where the coefficients u_k have to be characterized.
As asking those coefficient to be "simply" analytic in x is not sufficient, we have to introduce intermediary functions φ to studying u_k has an analytic function in the variable x and in the intermediary functions φ.
To conclude this talk, we will illustrate this idea in the case $p = 0$, that is a totally different problem (*study of a limit layer with an attractive slow curve*), which is needing one intermediary function :

$$\varphi(x) := e^{-x/\varepsilon}$$

With this method, we will retrieve the Asymptotic Combined Development.

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