

O-MINIMAL SPECTRA, INFINITESIMAL SUBGROUPS AND COHOMOLOGY

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By the work of various authors on some conjectures of Pillay, it is possible to associate in a canonical functorial way, to each definably compact group G in an o-minimal expansion of a field, a compact (real) Lie group G/G^{00} obtained as the quotient of G by its “infinitesimal subgroup” G^{00} , where the quotient is equipped with the “logic topology”. Our first result is that the functor $G \mapsto G/G^{00}$ sends exact sequences of definably compact groups into exact sequences of Lie groups. As a side product we characterize the infinitesimal subgroup G^{00} as the unique type-definable subgroup of bounded index which is divisible and torsion free (this was known in the abelian definably connected case). We then study the connections between the Lie group G/G^{00} and the o-minimal spectrum \tilde{G} of G . We prove that G/G^{00} is a topological quotient of \tilde{G} . We thus obtain a natural homomorphism from the cohomology of G/G^{00} to the (Čech-)cohomology of \tilde{G} . We show that under suitable hypothesis on G this homomorphism is an isomorphism and we give some arguments to support the conjecture that these hypothesis are always verified.

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