# Nonnegative Matrices

Judi McDonald
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Nonnegative matrix theory is an important area of linear algebra that has been built up from the Perron-Frobenius Theorem and has largely been driven by applications. This minisymposium brings together individuals with experience and interests in classical nonnegative matrix theory, as well as in a variety of generalizations and applications.

### Spectrally arbitrary patterns over finite fields.

E. J. Bodine, Washington State University
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*Fri 17:35, Room Pacinotti*

In this talk, we will explore zero-nonzero patterns over finite fields. In particular, we will examine patterns that demonstrate fundamental differences in the algebraic structure of different fields.

Joint work with J. J. McDonald (Washington State University)

### Sign patterns that require or allow power-positivity

Minerva Catral, Iowa State University, USA
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*Fri 16:45, Room Pacinotti*

A matrix $A$ is power-positive if some positive integer power of $A$ is entrywise positive. A matrix $A$ is eventually positive if $A^k$ is entrywise positive for all sufficiently large integers $k$. A characterization of sign patterns that require power-positivity is presented. It is also shown that a sign pattern $\mathcal{A}$ allows power-positivity if and only if $\mathcal{A}$ or $-\mathcal{A}$ allows eventual positivity.

Joint work with Leslie Hogben, Iowa State University, USA (lhogben@iastate.edu) & American Institute of Mathematics (hogben@aimath.org), D. D. Olesky, University of Victoria, Canada (dolesky@cs.uvic.ca), P. van den Driessche, University of Victoria, Canada (pvdd@math.uvic.ca)

### Matrix Functions Preserving Sets of Generalized Nonnegative Matrices

A. Elhashash, Drexel University, USA
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*Thu 12:15, Room Pacinotti*

We characterize matrix functions preserving several sets of generalized nonnegative matrices. These sets include PFn, the set of $n \times n$ real eventually positive matrices; and WPFn, the set of matrices $A \in \mathbb{R}^{n\times n}$ such that $A$ and its transpose has the Perron-Frobenius property. We also present necessary and sufficient conditions for a matrix function to preserve the set of $n \times n$ real eventually nonnegative matrices and the set of $n \times n$ real exponentially nonnegative. Moreover, we show that the only complex polynomials that preserve the set of $n \times n$ real exponentially nonnegative matrices are $p(z) = az + b$ where $a, b \in \mathbb{R}$ and $a \geq 0$.

Joint work with D. B. Szyld (Temple University)

### Nonnegative Jordan bases and characterization of eventually nonnegative matrices

D. Noutsos, University of Ioannina, Greece
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*Thu 11:00, Room Pacinotti*

For an $n \times n$ eventually nonnegative matrix $A$, the existence of a nonnegative Jordan basis of the eigenspace corresponding to the dominant eigenvalue is proven. This result is used to characterize when the matrix $hI + A$ is eventually nonnegative for all $h > 0$. Sufficient and necessary conditions are proven for this situation. Numerical examples are presented to illustrate and validate the theoretical results.

Joint work with M. J. Tsatsomeros (Washington State University)

### Sign Patterns that Allow Eventual Positivity

Dale Olesky, University of Victoria, Victoria, BC Canada
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*Fri 15:50, Room Pacinotti*

A real square matrix $A$ is *eventually positive* if there exists a positive integer $k_0$ such that for all $k \geq k_0$, $A^k$ is entrywise positive. Some necessary or sufficient conditions for a sign pattern to allow eventual positivity are given. It is shown that certain families of sign patterns do not allow eventual positivity, and that for $n \geq 2$, the minimum number of positive entries in an $n \times n$ sign pattern that allows eventual positivity is $n + 1$. All $2 \times 2$ and $3 \times 3$ sign patterns are classified as to whether or not the pattern allows eventual positivity. A $3 \times 3$ matrix is presented to demonstrate that the positive part of an eventually positive matrix need not be primitive, answering negatively a question of Johnson and Tarazaga.

### Computations with totally nonnegative and sign-regular matrices

J.M. Peña, University of Zaragoza, Spain
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*Thu 11:25, Room Pacinotti*

Totally nonnegative matrices are matrices with all their minors nonnegatives. They belong to the class of sign-regular matrices. We present recent advances of numerical methods for these classes of matrices. Methods for solving linear systems and methods concerning with their factorizations and related accurate computations are considered, as well as tests to recognize if a matrix belongs to these classes of matrices. Recent advances on localization results for their eigenvalues are also analyzed.

### Some constructive methods in the symmetric nonnegative inverse eigenvalue problem

H. Šmigoc, University College Dublin, Dublin
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*Fri 15:00, Room Pacinotti*

The question, which lists of complex numbers are the spectrum of some nonnegative matrix, is known as the nonnegative inverse eigenvalue problem (NIEP), and the same question posed for symmetric nonnegative matrices is called the symmetric nonnegative inverse eigenvalue problem (SNIEP). In the talk we will present some constructive methods applied to the SNIEP and discuss the effect of adding zeros to the spectrum in the SNIEP.

Joint work with T. J. Laffey(University College Dublin)

**Graphs, Patterns and Powers – From Nonnegative Matrices to Nonpowerful Ray Patterns**

Jeffrey Stuart, Pacific Lutheran University, Tacoma, Washington, USA
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*Fri 15:25, Room Pacinotti*

For positive powers of square matrices, which properties arise from the signed digraph and which depend on the relative magnitudes of the entries? There is a substantial body of research on this question, starting with the work on primitivity and imprimitivity for nonnegative matrices. Later, this study was extended to examining the zero-nonzero patterns of powers of real matrices and their connection to the signed digraph for the matrix. Viewed in terms of sign pattern classes, this led to the study of powerful sign patterns as well as the base and period of a powerful sign pattern. Subsequently, the results for powerful sign pattern classes were extended to results for powerful ray pattern classes, although there are some interesting differences between the behavior of sign patterns and ray patterns. Recently, interest has shifted to the behavior of powers of square, complex ray patterns that are not powerful, which is to say, for which the ray-signed digraph does not determine the ray patterns of powers of matrices in the qualitative class. For nonpowerful ray patterns, the concepts of base, power and primitivity break down in ways that have sometimes been overlooked in recent papers. We will discuss these ideas and some of the current directions of research.

**On the periodic stabilization of discrete-time positive switched systems**

Maria Elena Valcher, University of Padova, Italy
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*Thu 11:50, Room Pacinotti*

Positive switched systems typically arise to cope with two distinct modeling needs. On the one hand, switching among different models mathematically formalizes the fact that the system laws change under different operating conditions. On the other hand, the variables to be modeled may be quantities that have no meaning unless positive (temperatures, pressures, population levels, ...). Research interests in positive switched systems mainly focused on their stability properties [1], while stabilizability has been only marginally touched upon. In this talk we present some results about the stabilization of positive switched systems. In detail, we consider the class of discrete-time positive switched systems, described, at each time $t \in \mathbb{Z}_+$, by the first-order difference equation:

$$x(t+1) = A_{\sigma(t)} x(t), \tag{1}$$

where $x(t)$ denotes the $n$-dimensional state variable at time $t$, while $\sigma$ is a switching sequence, defined on $\mathbb{Z}_+$ and taking values in the finite set $\mathcal{P} = \{1, 2\}$. For each $i \in \mathcal{P}$, $A_i$ is an $n \times n$ positive matrix.

Assuming that both $A_1$ and $A_2$ are not Schur matrices, we provide conditions on the matrix pair $(A_1, A_2)$ that ensure that for every positive initial state $x(0) \in \mathbb{R}^n_+$, there is a switching law $\sigma : \mathbb{Z}_+ \to \mathcal{P}$ that leads the state trajectory $x(t) = A_{\sigma(t-1)} A_{\sigma(t-2)} \dots A_{\sigma(1)} A_{\sigma(0)} x(0)$ to zero as $t$ goes to $+\infty$. In particular, we show that every stabilizable positive switched system can be stabilized by means of a periodic switching law. Finally, we prove that, if a Schur convex combination of the matrices $A_1$ and $A_2$ can be found, it provides some information on the period and the "duty cycle" of these stabilizing periodic switching sequences.

[1] O. Mason and R.N. Shorten. Quadratic and copostive Lyapunov functions and the stability of positive switched linear systems. In Proceedings of the American Control Conference (ACC 2007), pp. 657–662, New York, 2007.

Joint work with Ettore Fornasini (University of Padova)

**Spectrally arbitrary zero-nonzero patterns**

A. A. Yielding, Eastern Oregon University, La Grande, OR
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*Fri 17:10, Room Pacinotti*

In this talk we establish the lower bound of $2n-2$ for the number of zero entries an $n \times n$ irreducible zero-nonzero pattern that is not spectrally arbitrary and contains at least two nonzero entries along the diagonal.

Joint work with J. J. McDonald (Washington State University)