Matrix Inequalities — In Memory of Ky Fan

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The purpose of this symposium is to stimulate researches in the area of matrix and operator inequalities and to provide an opportunity for mathematicians in the field to exchange ideas and share most recent developments and information.

On distance measures between positive semidefinite matrices and their applications in quantum information theory

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Mon 11:00, Auditorium

The theory of matrix inequalities has many applications in quantum information theory. In this talk we consider a number of distance measures between quantum states that are commonly used. Many problems in quantum information theory depend on finding useful relationships between these distance measures. These relationships can be generalised to matrix inequalities for positive semidefinite matrices. We give an overview of these inequalities, some of which are straightforward consequences of known inequalities, while others have been open problems for a long time, e.g. an inequality concerning the generalisation of the Chernoff distance between two probability distributions to quantum states. In addition, a number of conjectured inequalities are presented, too. In particular, we present some recent progress on a conjectured inequality involving N positive semidefinite matrices that would fill the only remaining gap in an argument extending the quantum Chernoff distance to N quantum states.

Joint work with Milan Mosonyi (Budapest University of Technology and Economics)

Matrix subadditivity inequalities

Jean-Christophe Bourin, Université de Franche-Comté, France
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Mon 11:25, Auditorium

This talk surveys several recent results of functional analytic spirit in matrix analysis. Most of these results are subadditivity inequalities for symmetric norms (or unitarily invariant norms) and concave functions of operators. In case of normal operators, this leads to some estimates for partitioned matrices.

$-(\sigma_1 - \sigma_2)^4 \geq 0$

Roger A. Horn, University of Utah, Salt Lake City, Utah USA
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Mon 11:50, Auditorium

In 1961, D. C. Youla discovered a block upper triangular form to which any square complex matrix A can be reduced by a unitary congruence, that is, $A \to UAU^T$, in which U is unitary. We revisit Youla's form and describe a canonical form for its diagonal blocks. The inequality in the title plays a key role in identifying the diagonal blocks associated with real negative eigenvalues of $A\bar{A}$.

Jensen matrix inequalities and direct sums

Fuad Kittaneh, University of Jordan, Jordan
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Mon 12:15, Auditorium

Let A, X and Y be n-by-n complex matrices such that A is positive semi-definite and X, Y are contractions. We prove that if f is an increasing convex function on $[0, \infty)$ such that $f(0) \leq 0$, then the eigenvalues of $f(|X^*AY|)$ are dominated by those of $X^*f(A)X \oplus Y^*f(A)Y$. Several related results are considered.

Joint work with J.-C. Bourin (Université de Franche-Comté) and O. Hirzallah (Hashemite University)

Operator Radii and Unitary Operators

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Tue 11:00, Auditorium

Let $\rho \geq 1$ and $w_\rho(A)$ be the operator radius of a linear operator A. Suppose m is a positive integer. It is shown that for a given invertible linear operator A acting on a Hilbert space, one has $w_\rho(A^{-m}) \geq w_\rho(A)^{-m}$. The equality holds if and only if A is a multiple of a unitary operator.

Joint work with Tsuyoshi Ando, Professor Emeritus, Hokkaido University.

Perturbation of Partitioned Hermitian Generalized Eigenvalue Problem

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We are concerned with Hermitian positive definite generalized eigenvalue problem $A - \lambda B$ for partitioned

$$A = \begin{pmatrix} A_{11} & \\ & A_{22} \end{pmatrix}, \quad B = \begin{pmatrix} B_{11} & \\ & B_{22} \end{pmatrix},$$

where both A and B are Hermitian and B is positive definite. Bounds on how its eigenvalues varies when A and B are perturbed by Hermitian matrices. These bounds are generally of linear order with respect to the perturbations in the diagonal blocks and of quadratic order with respect to the perturbations in the off-diagonal blocks. The results for the special case of no perturbations in the diagonal blocks can be used to bound the changes of eigenvalues of a Hermitian positive definite generalized eigenvalue problem after its off-diagonal blocks are dropped, a situation occurs frequently in eigenvalue computations.

Stewart and Sun (1990) observed that different copies of a multiple eigenvalue for the generalized eigenvalue problem may behave very differently. Recently, Nakatsukasa (2009) successfully obtained quantitative estimates to explain the behavior. In this talk, we will present different estimates.

Supported in part by the National Science Foundation under Grant No. DMS-0702335 and DMS-0810506.

One horse racing story, two card games, and three matrix theorems

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Tue 11:50, Auditorium

Motivated by a horse racing story in ancient China, we consider two card games. Let D_1 and D_2 be two diagonal matrices whose diagonal entries correspond to the card values of the two players of the card games. There is a correspondence between the outcomes of the first card game and the possible inertia of a matrix of the form $P^tD_1P - Q^tD_2Q$, where P and Q are permutation matrices. It turns out that there is also a correspondence between the outcomes of the second card game and the possible inertia of a matrix of the form $U^*D_1U - V^*D_2V$, where U and V are complex unitary (or real orthogonal) matrices. Using the simple strategy in the ancient story, we describe all the possible outcomes of the card games and the inertia of the corresponding matrices $P^tD_1P - Q^tD_2Q$ and $U^*D_1U - V^*D_2V$. Related problems and results are also mentioned.

Joint work with Chi-Kwong Li (Department of Mathematics, The College of William and Mary, U.S.A.)

Loewner matrices and matrix convexity

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Tue 12:15, Auditorium

In this talk, our results on Loewner matrices $\left[\frac{f(p_i)-f(p_j)}{p_i-p_j}\right]$ in [1] and [2] will be presented. Moreover, related results are to be reported.

[1] R. Bhatia, T. Sano, Loewner matrices and operator convexity, Math. Ann. 344 (2009), no. 3, 703–716.
[2] R. Bhatia, T. Sano, Positivity and conditional positivity of Loewner matrices, to appear in Positivity.

Inequalities in Construction of Higher Rank Numerical Ranges

Raymond Nung-Sing Sze, Department of Applied Mathematics, The Hong Kong Polytechnic University, Hung Hom, Hong Kong
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Thu 11:00, Auditorium

Given a positive integer k, the higher rank numerical range $\Lambda_k(A)$ of a matrix A is the set of complex λ such that for some rank k projection P we have $PAP = \lambda P$. It has been shown that the set $\Lambda_k(A)$ can be constructed by infinitely many inequalities of A. In particular, only finite many inequalities are needed if A is normal. In this talk, we revisit and demonstrate these constructions and related results.

Joint work with H.L. Gau (National Central University), C.K. Li (College of William and Mary), and Y.T. Poon (Iowa State University)

Determinant and Pfaffian of sum of skew symmetric matrices

Tin-Yau Tam, Auburn University, USA
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Thu 11:25, Auditorium

We completely describe the determinants of the sum of orbits of two real skew symmetric matrices, under similarity action of orthogonal group and the special orthogonal group respectively. We also study the Pfaffian case and the complex case. Inequalities are obtained.

Joint work with Mary Clair Thompson (Auburn University)

Revisiting a Permanent Conjecture on Positive Semidefinite Matrices

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Thu 11:50, Auditorium

We will revisit the permanent conjecture $per(A \circ B) \leq per(A)per(B)$ with the maximizing matrix approach, where A and B are positive semidefinite matrices and $A \circ B$ is the Hadamard product of A and B.

The Equality Cases for the Inequalities of Oppenheim and Schur for Positive Semi-definite Matrices

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Thu 12:15, Auditorium

In matrix inequality theory, the inequalities of Oppenheim and Schur for positive semi-definite matrices are well known. In this talk, we investigate under which conditions the Hadamard product of two positive semi-definite matrices are singular. These results are used to give necessary and sufficient conditions for equality in the inequalities of Oppenheim and Schur for positive semi-definite matrices.

[1] R. B. Bapat and T. E. S. Ragharan, *Nonnegative Matrices and Applications,* Cambridge University Press, 1997.
[2] A. Oppenheim, Inequalities connected with definite Hermitian forms, *J. London Math. Soc.* 5 (1930), 114-119.
[3] X.-D. Zhang, and C.-X. Ding, The equality cases for the inequality of Oppenheim and Schur for positive semi-definite matrices, Czechoslovak Mathematical Journal, 59 (134) (2009), 197-206.

Joint work with Chang-Xing Ding (Shanghai Jiao Tong University)