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Advances in Nonlinear Problems from Materials Science and Shape Optimization

Organizing Committee : Giuseppe Buttazzo, Maria Stella Gelli, Matteo Novaga

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Abstracts

Roberto Alicandro: *Interactions beyond nearest neighbours and rigidity of discrete energies*

Abstract. I will analyse the rigidity of discrete energies where at least nearest and next-to-nearest neighbour interactions are taken into account. My purpose is to examine the role of interactions beyond nearest neighbours in penalising changes of orientation and how, to some extent, they may replace the positive-determinant constraint that is usually required when only nearest neighbours are accounted for. Following the same approach, I will also present the asymptotic analysis of a discrete model for nanowires.

Luigi Ambrosio: *Some remarks on metric measure structures and their convergence*

Abstract. In the talk I will review some aspects of the theory of Sobolev spaces in metric measure spaces. For the sake of illustration, I will describe the counterpart of these results for elliptic quadratic forms with measurable coefficients in R^n and the G and Γ convergence for a distinguished class of energies, the so-called Cheeger energies.

Lorenzo Brasco: *Regularity for the fractional p-Laplacian and a new transport problem*

Abstract. We present some regularity results for weak solutions of nonlocal variants of the $p-$Laplace equation. The main instance of such equations is the first variation of an Aronszajn–Gagliardo–Slobodeckij seminorm. In particular, we show a higher differentiability result for solutions, recently obtained in collaboration with Erik Lindgren (KTH). We discuss some connections of these equations to a new Optimal Transport problem with congestion effects and present some open issues.

Juan Casado Diaz: *The homogenization of the wave equation with oscillating coefficients*

Abstract. For a wave equation with periodic coefficients of small period, it is well known that the corresponding limit equation, when the period tends to zero, is obtained by replacing the diffusion matrix by the elliptic homogenized one. However, the elliptic corrector does not give a corrector for the wave problem. We show that even if the coefficient matrix does no depend on the time variable, the solution oscillates in both space and time. Moreover, contrarily to the homogenization of elliptic or parabolic problems, the corrector for the wave equation is non-local and can depend on the initial and boundary conditions.

Guillaume Carlier: *Entropic regularization of optimal transport*

Abstract. The Monge-Kantorovich mass transport problem is an infinite-dimensional (or very high dimensional after discretization) linear programming problem which can be difficult to solve numerically. In this talk, I will discuss the entropic regularization of such problems which actually goes back to Schrdinger in the early 1930s both from the numerical and theoretical points.

Thierry Champion: *A new class of costs for optimal transport problems*

Abstract. In optimal mass transport theory, many problems can be written in the Monge-Kantorovich form

$$(MK) \qquad \inf\left\{\int_{X\times Y} c(x,y)\,d\gamma \ : \ \gamma\in\Pi(\mu,\nu)\right\}$$

where μ,ν are given probability measures on X,Y and $c: X\times Y\to[0,+\infty]$ is a cost function. Here the competitors are probability measures γ on $X\times Y$ with marginals μ and ν respectively (the set $\Pi(\mu,\nu)$ being the set of these transport plans). In the particular case where an optimal transport plan $\gamma\in\Pi(\mu,\nu)$ is carried by the graph of a map $T: X\to Y$, i.e. if

$$\langle\gamma,\varphi\rangle=\int_X \varphi(x,Tx)\,d\mu \quad , \quad T^\sharp\mu=\nu\ ,$$

then T solves the original Monge problem: $\inf\left\{\int_X c(x,Tx)\,d\mu \ : \ T_\sharp\mu=\nu\right\}$. Here we are interested in a different case. Indeed in some applications to economy or in probability theory, it can be interesting to favour optimal plans which are non associated to a single valued transport map $T(x)$. The idea is then to consider, instead of $T(x)$, the family of conditional probabilities γ^x such that

$$\langle\gamma,\varphi\rangle=\int_X\left(\int_X \varphi(x,y)d\gamma^x(y)\right)\,d\mu\ ,$$

and to incorporate in problem (MK) an additional cost over γ^x as follows

$$(MK_2) \qquad \inf\left\{\int_{X\times Y} c(x,y)\,d\gamma+\int_X G(x,\gamma^x)\,d\mu \ : \ \gamma\in\Pi(\mu,\nu)\right\}\ ,$$

being $G:(x,p)\in X\times\mathcal{P}(X)\to[0,+\infty]$ a given non linear function. In this talk I will

describe some recent results concerning problem (MK_2) (existence, duality principle, optimality conditions) and focus on specific examples where $X = Y$ and X is a convex compact subset of $\mathbb{R}^d$. This is a joined work with G. Bouchitté and J.-J. Alibert

(Université de Toulon).

bigskip

Marco Cicalese: *On global and local minimizers of prestrained thin elastic rods*

Abstract. We study the stable configurations of a thin 3-dimensional weakly prestrained rod subject to a terminal load as the thickness of the section vanishes. By Gamma-convergence we derive a limit 1-dimensional theory and show that isolated local minimizers of the limit model can be approached by local minimizers of the 3-dimensional model. In the case of isotropic materials and for two-layers prestrained 3-dimensional models the limit energy further simplifies to that of a Kirchhoff rod-model of an intrinsically curved beam. In this case we study global and local stability of straight and helical configurations. Some simple simulations help us to compare our results with real experiments.

Matteo Focardi: *An epiperimetric inequality for the thin obstacle problem*

Abstract: We report on an epiperimetric inequality for the thin obstacle problem, extending the pioneering results by Weiss on the classical obstacle problem (Invent. Math., 138 (1999), no.1, 23–50). This inequality provides the means to study the rate of converge of the rescaled solutions to their limits, as well as the regularity properties of the related free boundary. This is joint work with E. Spadaro (MPI Leipzig).

Ilaria Fragalà: *Boundary value problems for the infinity Laplacian: regularity and geometric results*

Abstract. We discuss regularity and geometric results for boundary value problems where the operator is the infinity Laplacian, or its normalized version. In particular, we focus our attention on the homogeneous Dirichlet problem with constant source term, and on a related Serrin-type overdetemined problem. The talk is based on some recent joint works with Graziano Crasta, University of Roma "La Sapienza".

Alessandro Giacomini: *A relaxed framework for the optimization of Robin eigenvalues*

Abstract. We propose an SBV approach to the optimization of higher eigenvalues of the Robin-Laplacian. In particular we find a variational problem on a suitable space

of vector valued functions of SBV-type which can be considered as a relaxation of the classical shape optimization problem. This is a joint work with Dorin Bucur

Andrea Malchiodi: *Embedded Willmore tori in three-manifolds with small area constraint*

Abstract. While there are lots of contributions on Willmore surfaces in the three-dimensional Euclidean space, the literature on curved manifolds is still relatively limited. One of the main aspects of the Willmore problem is the loss of compactness under conformal transformations. We construct embedded Willmore tori in manifolds with a small area constraint by analysing how the Willmore energy under the action of the Mbius group is affected by the curvature of the ambient manifold. The loss of compactness is then taken care of using minimisation arguments or Morse theory.

Emanuele Paolini: *Minimal planar bubble clusters composed by four equivalent regions*

Abstract. A bubble cluster is a union of disjoint regions with prescribed volumes. We are interested in clusters with minimal interface area, thus generalizing the isoperimetric problem (which is the particular case of a single region). The case of two regions has been completely solved in any space dimension and the case of three regions has been completely solved in the planar case. In general it is an open problem whether minimizers are composed by connected regions. We solve the problem in the case of four planar regions with equal area and find the only possible topology of minimizers. The results have been obtained in the PhD thesis of Andrea Tamagnini (available on cvgmt.sns.it).

Matthias Roger: *A stochastic mean curvature flow for graphs*

Abstract. We study a stochastically perturbed mean curvature flow for graphs over the two-dimensional unit-cube subject to periodic boundary conditions. The stochastic perturbation is a one dimensional white noise acting uniformly in all points of the surface in normal direction. We establish the existence of a weak martingale solution. The proof is based on energy methods and therefore presents an alternative to the stochastic viscosity solution approach.

Lucia Scardia: *Convergence of interaction-driven evolutions of dislocations*

Abstract. It is well known that the plastic, or permanent, deformation of a metal is caused by the movement of curve-like defects in its crystal lattice. These defects are

called dislocations. What is not yet clear is how to use this microscale information to make theoretical predictions at the continuum scale.

Motivated by this, M.G. Mora, M. Peletier and I considered a system of interacting dislocations and studied the convergence of the evolution of the corresponding empirical measures in the limit of many dislocations. In the continuum limit we obtained an evolution law for the dislocation density.

In this talk I will present this result and discuss its limitations and further extensions towards more realistic and complex systems.

Bozhidar Velichkov: *Regularity of optimal sets for spectral functionals*

Abstract. We consider the variational problem

$$\min\left\{\sum_{j=1}^{k}\lambda_j(\Omega)\ :\ |\Omega|\le 1,\ \Omega\subset\mathbb{R}^d\right\},$$

in which the variable is the domain $\Omega\subset\mathbb{R}^d$, $|\cdot|$ is the Lebesgue measure and the cost functional is the sum of the first k Dirichlet eigenvalues on Ω. We prove that the optimal sets have $C^{1,\alpha}$ regular boundary up to a set of zero $\mathcal{H}^{d?1}$-measure. This is strongly related to the regularity of the free boundary $\partial\{|U|>0\}$ of the local minima of the functional

$$H^1_{loc}(\mathbb{R}^d;\mathbb{R}^k)\ni U\mapsto\int|\nabla U|^2\,dx+|\{|U|>0\}|,$$

to which we will dedicate most of our attention.