

PA and FA or: On some properties of automorphism groups of arithmetically saturated models of arithmetic

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A group G has the property FA if whenever it acts without inversions on a tree T , there is a vertex of T fixed by all g in G . If G is not finitely generated, by a theorem of Serre, this is equivalent to the following three conditions:

1. G is not a free product with amalgamation.
2. The infinite cyclic group is not a homomorphic image of G .
3. G has uncountable cofinality.

THEOREM. A countable recursively saturated model of true arithmetic (TA) has property FA iff it is arithmetically saturated.

I will not talk about the algebraic content of this result, instead I will explain some model theory of PA behind it. In particular, I will outline an argument, due to Jim Schmerl, which implies the TA can be replaced with PA in the theorem, provided a certain conjecture concerning chromatic numbers of Cartesian products of graphs is true.

BIBLIOGRAPHY

- [1] R. Kossak, A. Ivanov, Automorphism group actions on trees, *Math. Log. Quart.*, 50, 71-76, (2004).
- [2] R. Kossak, J. Schmerl, Arithmetically saturated models of arithmetic, *Notre Dame J. Formal Logic*, 36, 531-546 (1995).
- [3] D. Macpherson, S. Thomas, Comeagre conjugacy classes and free products with amalgamation, to appear.
- [4] J. Schmerl, Generic automorphisms and graph coloring, to appear.