

Definable groups in o-minimal structures

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We are interested in definable groups in an o-minimal structure M , namely groups whose underlying set is definable and with a definable group operation. Such groups have been studied by many authors and the results so far obtained suggest a close analogy between definable groups and Lie groups. In particular it follows from the results of A. Pillay (1988) that if the underlying set of the o-minimal structure is the real line, then every definable group is indeed a Lie group, while Y. Peterzil, A. Pillay e S. Starchenko (2000) obtained matrix representation theorems for definable groups which confirm the analogy with Lie groups. Recently Pillay (2003) stated a conjecture which, if solved positively, would greatly clarify the relationship between Lie groups and definable groups. As a partial answer to the conjecture we prove, in collaboration with M. Otero, Y. Peterzil and A. Pillay, that it is possible to associate in a canonical (and non-trivial) way to every definable groups G a compact Lie group G/G^{00} . Here G^{00} is the smallest “type definable subgroup of bounded index” (whose existence is part of the result) and G/G^{00} has a suitable “logic topology” studied by Lascar and Pillay. One would expect that if G is definably compact then G should resemble G/G^{00} very closely (say it is elementary equivalent).

A. Berarducci, M. Otero, Y. Peterzil, A. Pillay, “A descending chain condition for groups definable in o-minimal structures”, RAAG Preprint n.103 (15p.; 2004, May 3).http://www.uni-regensburg.de/Fakultaeten/nat_Fak_I/RAAG/