USEFUL AXIOMS

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I overview several aspects of forcing axioms which (in my eyes) give solid mathematical arguments explaining why these axioms are so useful in establishing new (consistency) results and/or theorems.

The first aspect outlines that forcing axioms are natural strengthenings not only of Baire?s category theorem, but also of the axiom of choice (these are two of the most useful non-constructive principles in mathematics), and also strengthenings of most large cardinal axioms (at least for cofinally many of them).

The second aspect outlines that Shoenfield?s absoluteness, Cohen?s forcing theorem, and Los theorem for standard ultrapowers of a first order structure by a non principal ultrafilter are all specific instances of a more general form of Los theorem which can be declined for what I call boolean ultrapowers.

The third aspect outlines how strong forcing axioms and Woodin?s generic absoluteness results are two sides of the same coin and will try to explain how stronger and stronger forms of generic absoluteness can be obtained by asserting stronger and stronger forcing axioms. In this context category theoretic ideas start to play a role and we are led to analyze forcings whose conditions are (certain classes of) forcing notions and whose order relation is given by (certain classes of) complete embeddings. There is a surprising analogy between the theory of these class forcings, the theory of towers of normal ideals, and many of the classical arguments yielding generic absoluteness results.

For what concerns the first two aspects of my talk, I do not claim authorship of essentially none of the result I will be talking about, nonetheless it is hard to attribute correctly the relevant results.

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