

On the Newton method for the matrix p th root

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We consider the Newton iteration for computing the principal matrix p th root, which is rarely used in the application for the bad convergence and the poor stability.

We analyze the convergence conditions. In particular it is proved that the method converges for any matrix A having eigenvalues with modulus less than 1 and with positive real part.

Based on these results we provide a general algorithm for computing the principal p th root for any matrix A having no nonpositive real eigenvalues.

The algorithm has a quadratic convergence, is numerically stable and has a cost of $O(n^3 \log p)$ ops per step.