On the regularizing power of multigrid-type algorithms

Marco Donatelli* Stefano Serra-Capizzano †

September 8, 2004

Abstract

We consider the deblurring problem of noisy and blurred images in the case of space invariant point spread functions. We combine an algebraic multigrid previously defined ad hoc for structured matrices related to space invariant operators (Toeplitz, circulants, trigonometric matrix algebras etc.) and the classical geometric multigrid studied in the partial differential equations context. The resulting technique is parameterized in order to have more degrees of freedom: a simple choice of the parameters (not in charge to the users!) allows to devise a powerful regularizing method whose features are the following:

1. it is used with early stopping (as the CG, CGNE, and the Landweber method),
2. its cost per iteration is comparable with the classical ones (CG, Landweber, CGNE),
3. the minimal relative error with respect to the true image is sensibly lower with regard to all the best known techniques directly applied to the system $A\mathbf{f} = \mathbf{g}$ (Riley, CG, PCG, etc.) with optimal parameters and the associated curve of the relative errors with respect to the iterations is "flatter",
4. the minimal relative error with respect to the true image is comparable or slightly lower with regard to all the best known techniques for the normal equations $A^T A\mathbf{f} = A^T\mathbf{g}$ (Tikhonov, CGNE, PCGNE, Landweber etc) with optimal parameters and the convergence is extremely faster.

Therefore, as a consequence of 3, the choice of the exact iteration where to stop is less critical than in other regularizing iterative methods while, as a consequence of 4, we can choose multigrid procedures which are much more efficient than classical techniques without losing accuracy in the restored image. Several numerical experiments show the effectiveness of our proposals.

*Dipartimento di Fisica e Matematica, Università dell'Insubria - Sede di Como, Via Valleggio 11, 22100 Como, Italy (marco.donatelli@uninsubria.it).

†Dipartimento di Fisica e Matematica, Università dell'Insubria - Sede di Como, Via Valleggio 11, 22100 Como, Italy (stefano.serrac@uninsubria.it, serra@mail.dm.unipi.it).