Entrance test

FCS UNIPI - Math 0 class

September 9^{th}, 2021

Instructions

Write not only the solutions **but also the reasoning and the significant steps for every exercise**. If possible, use plain white paper. The allotted time for the test is 2 hours. When you have completed the test please take photos of the solutions, in the correct order if possibile, and send them with **one** email to the address `caboara@dm.unipi.it` using the subject **FCS-your family name**. You will receive your grade as soon as possible.

The grade only use will be to decide if the Math 0 class could be useful to you.

1. Compute $15/18 - 6/16$.
2. Is it true that $\sqrt{2} + 2 + \sqrt{3} > 1 + \sqrt{5}$?
3. Compute $\left(2\sqrt{2} + \frac{\sqrt{3}}{\sqrt{6}}\right)^2$.
4. Solve, considering existence conditions, the equation
$$\frac{x\sqrt{x^2+1}}{x^2} = 0$$
5. Solve, considering existence conditions, the equation
$$\frac{(x^2-16)(x-1)}{x-4} = 0$$
6. Simplify the fraction $\dfrac{a^2 - b^2 + a - b}{a - b}$.
7. Find the greater common divisor and the least common multiple of the integers $105, 110$.

8. Find the greater common divisor and the least common multiple of the polynomials $x^2 - 1, \quad x^2 + 3x + 2$.

9. Solve the equation $27x - 18 = 0$.

10. Solve the equation $x^2 - x - 6 = 0$.

11. Solve the equation $5 - 3x > x$.

12. Solve the inequality $x^2 - x - 6 < 0$.

13. Solve, considering existence conditions, the inequality

$$\frac{x+2}{x-1} \leq 0$$

14. Simplify the fraction $\dfrac{x^2 - 2x - 15}{x^2 - 4x - 21}$.

15. Draw on the Cartesian plane the line for $A : (1, 1)$, $B : (1, 3)$.

16. Draw on the Cartesian plane the parabola $f(x) = x^2 + 4x - 5$.

17. Solve graphically the inequality $2^x + x - 1 > 0$.

18. Given the function $f(x) = \sqrt{x^2 - 1}$, find $f(1)$ and $f^{-1}(0)$.

19. Simplify and calculate the expression

$$\left[\left(2 - \frac{3}{4}\right) \cdot \frac{6}{15}\right] \cdot \frac{1}{2} + \left(\frac{1}{2} + \frac{3}{8}\right) - \left(\frac{7}{6} - \frac{9}{8}\right)$$

20. Draw on the Cartesian plane the triangle ABC, where

$$A = (0, 0), \quad B = (0, 2), \quad C = (1, 0)$$

Find the area and the perimeter of the triangle.

21. Draw on the Cartesian plane the triangle ABC, where

$$A = (-1, 2), \quad B = (0, 5), \quad C = (2, 2)$$

Find the area and the perimeter of the triangle.

22. Solve, considering existence conditions, the inequality $\dfrac{1}{x+1} \geq \dfrac{1}{x^2+1}$.

23. Compute the area and perimeter of the circle with center in $C : (0, 2)$ and that intersects the line $x = 0$ at the origin $O : (0, 0)$.

24. Put in rational standard form the fraction $\dfrac{\sqrt{2}}{3-\sqrt{2}}$.

25. Put in rational standard form the fraction $\dfrac{1}{1+\sqrt[3]{3}}$.