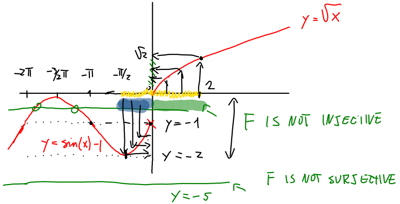
Ex $\quad F:\mathbb{R}\to\mathbb{R}$ $\qquad$ E.F.

$$x \rightsquigarrow \begin{cases} \sin(x)-1 & x<0 \\ \sqrt{x} & x\geq 0 \end{cases}$$



$F(0)=0$

$F(-\pi)=-1$

$$F([-\pi/2,2]) = F([-\pi/2,0)) \cup F([0,2]) =$$

F is ↑ in $[-\pi/2,0)$ $\quad$ ⇐ F is ↑ in $[0,+\infty)$

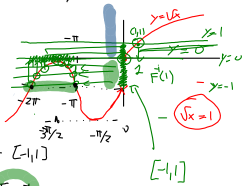
$$[F(0),F(2)] = [0,\sqrt{2}]$$

$$[F(-\pi/2),F(0)) = [-2,-1)$$

$$\overset{-1}{F}(0),\ \overset{-1}{F}(1),\ \overset{-1}{F}([-1,1])$$

$$\{0, -\tfrac{3}{2}\pi-2k\pi\}$$

$$-\tfrac{3}{2}\pi+2k\pi$$

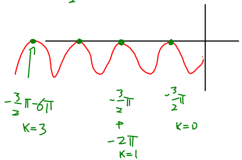


COUNTERIMAGE OF $[-1,1]$

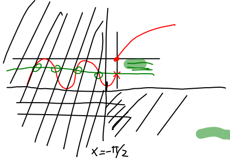
$$\overset{-1}{F}([-1,1])$$
$$\|$$
$$\overset{-1}{F}([-1,0]) \cup \overset{-1}{F}([0,1]) = [0,1]$$
$$\|$$
$$[-2\pi,-\pi]-2k\pi \quad k\in\mathbb{N}$$
$$\|$$
$$[-2\pi-2k\pi, -\pi-2k\pi] \quad k\in\mathbb{N}$$

$k=2$

$-\frac{3}{2}\pi-4\pi$



$$-\tfrac{3}{2}\pi-2k\pi \qquad k\in\mathbb{N}$$
$$\|$$
$$\{-\tfrac{3}{2}\pi-2k\pi \mid k\in\mathbb{N}\}$$
$$\|$$
$$\{-\tfrac{3}{2}\pi+2k\pi \mid k\in-\mathbb{N}\}$$



$F : \mathbb{R} \to \mathbb{R}$

$x \rightsquigarrow \begin{cases} \sin(x) - 1 & x < 0 \\ \sqrt{x} & x \geq 0 \end{cases}$

$F' : [0, +\infty) \to [0, +\infty)$

$F^{(IV)} : [3, +\infty) \to [\sqrt{3}, +\infty)$

$x \rightsquigarrow \sqrt{x}$

NOT INJ $F'' : [-\pi/2, +\infty) \to [-2, +\infty)$

$F''' : [-\pi/2, +\infty) \to [-2, -1) \cup [0, +\infty)$

$y = \sin(x) - 1$

$\sin(x) = y + 1$

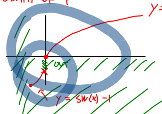
$\arcsin(\sin(x)) = \arcsin(y+1)$

$\Vert$

$x = \arcsin(y+1)$

$x \rightsquigarrow \begin{cases} \arcsin(x+1) & x \in [-\pi/2, 0) \\ x^2 & x \in [0, +\infty) \end{cases}$

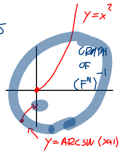
GRAPH OF F'''



INVERSES

$\leftrightarrow$ $y = x^2$

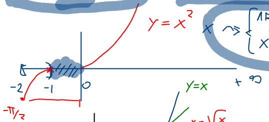
$\leftrightarrow$ $y = \arcsin(x+1)$

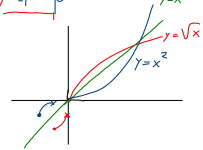


$F''' : [-\pi/2, +\infty) \to [-2, -1) \cup [0, +\infty)$

$(F''')^{-1} : [-2, -1] \cup [0, +\infty) \to [-\pi/2, +\infty)$

$x \rightsquigarrow \begin{cases} \arcsin(x+1) & x \in [-2, -1) \\ x^2 & x \in [0, +\infty) \end{cases}$





BLUE : INVERSE

IS THE MIRROR IMAGE OF

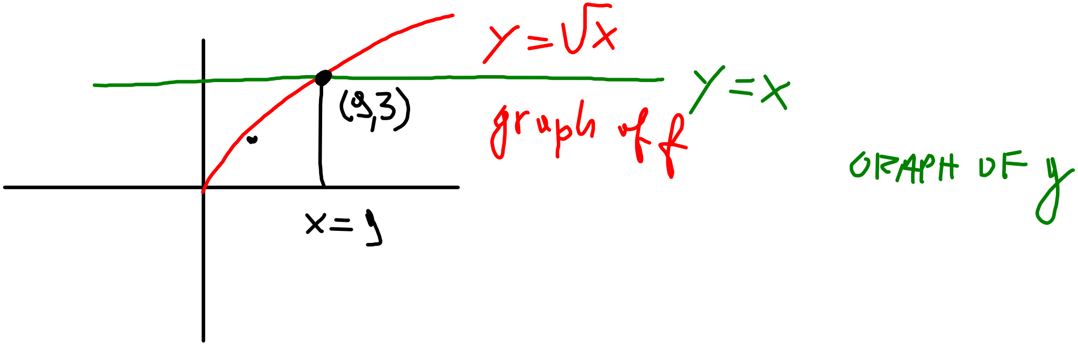
F''' WITH RESPECT TO

$y = x$

Ex

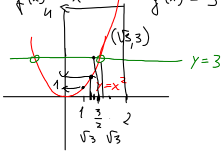
$$f : \mathbb{R}_0^+ \to \mathbb{R} \qquad\qquad g : \mathbb{R} \to \mathbb{R}$$

$$x \mapsto \sqrt{x} \qquad\qquad\qquad x \mapsto 3$$



INTERSECTION $\sqrt{x} = 3 \quad \Rightarrow \quad x = 9$

Ex $f(x) = x^2$ $g(x) = 3$



$x^2 = 3$

$x = \pm\sqrt{3}$

$\sqrt{3} = 1.73\ldots\ldots$

$1 < \sqrt{3} < 2$ OK

$\frac{3}{2} < \sqrt{3} < 2$ OR $\cancel{1 < \sqrt{3} < \frac{3}{2}}$?

$f\left(\frac{3}{2}\right) > 3$?

$\left(\frac{3}{2}\right)^2 > 3$? $\iff \frac{9}{4} > 3$ FALSE

$\frac{3}{2} < \sqrt{3} < 2$

WHAT IS THE MIDDLE POINT ?

$$\frac{3/2 + 2}{2} = \frac{3+4}{4} = \frac{7}{4}$$

$f\left(\frac{7}{4}\right) > 3$?

$3.065 = \frac{49}{16} > 3 \Rightarrow \underset{\overset{\shortparallel}{1.5}}{\frac{3}{2}} < \sqrt{3} < \underset{}{\frac{7}{4}} = 1.75$

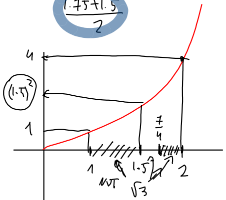
$\sqrt{3}$ SOMEWERE

1.5 —|— 1.75

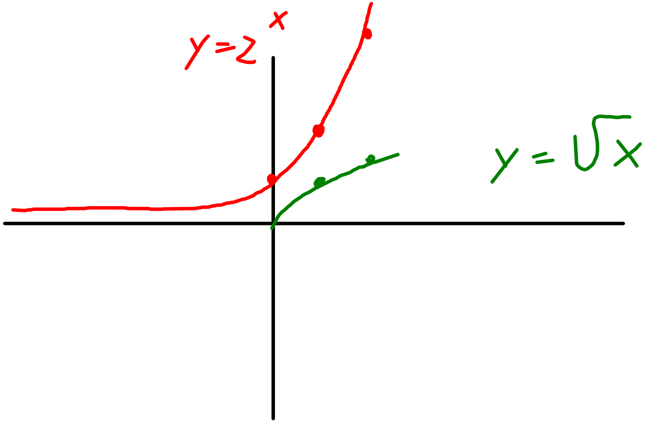
$$\frac{1.75 + 1.5}{2}$$

$\sqrt{3} \in \overbrace{[1.5, 1.75]}^{0.25}$

$$\sqrt{3} = \frac{1.75 + 1.5}{2} \pm (0.125)$$



$f(x) = 2^x$ $\quad$ $g(x) = \sqrt{x}$

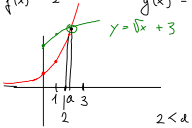


$f(x) = g(x)$

$2^x = \sqrt{x}$

NO SOLUTION

$f(x) = 2^x$ $\qquad g(x) = \sqrt{x} + 3$



s.t. $2^{\alpha} = \sqrt{\alpha} + 3$

$\frac{5}{2} = 2.5$

$2 < \alpha < 3$

$\boxed{2 < \alpha < 5/2}$ OR $\boxed{5/2 < \alpha < 3}$?

$f(5/2) < g(5/2)$?

$2^2\sqrt{2} = \sqrt[2]{2^5} = 2^{5/2} < \sqrt{5/2} + 3 \simeq 4.58$ NOT

$\overset{\shortparallel}{4\sqrt{2}}$

WE KNOW $f(5/2) > g(5/2)$

$\Downarrow$

$2 < \alpha < 5/2$

$2 < \alpha < 9/4$ OR $9/4 < \alpha < 5/2$?

MIDDLE POINT 2, 5/2

$\frac{\frac{5}{2} + 2}{2} = \frac{5+4}{4} = 9/4$

$f(9/4) < g(9/4)$?

$\sqrt[4]{2^9} = 2^{9/4}$ $\qquad \sqrt{9/4} + 3 = \frac{3}{2} + 3 = \frac{3+6}{2} = \frac{9}{2} = 4.5$

$4\sqrt[4]{2} = 4.75$ NOT THE CASE!

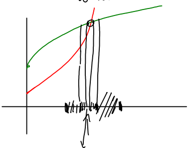
$f(9/4) > g(9/4) \Rightarrow 2 < \alpha < 9/4$

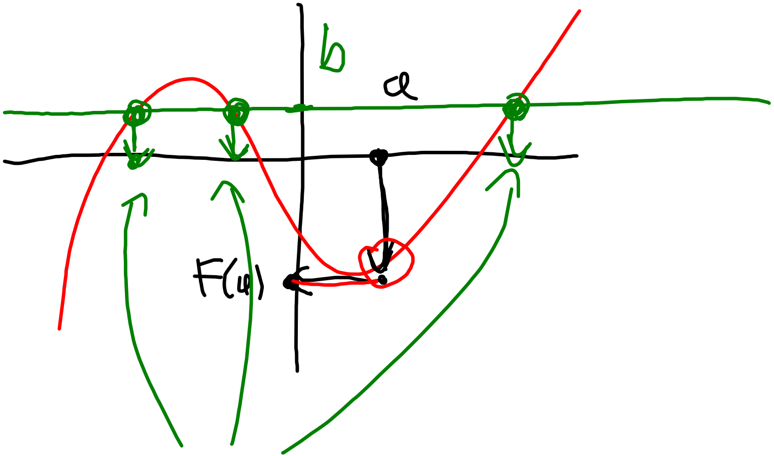
$9/4 = 2.25$

$\alpha \in (2, 2.25)$

$\alpha = \underbrace{\frac{2.25+2}{2}}_{\text{MIDDLE POINT}} \pm \frac{(2.25-2)}{2}$

$\alpha = \underbrace{2.125}_{\text{VALUE OF INTERSECTION}} \pm \underbrace{0.125}_{\text{ERROR}}$





$F^{-1}(b)$

$F: A \to B \quad b \in B$

$$F^{-1}(b) = \{a \in A \mid F(a) = b\}$$