FCS
Math: Functions
Exercises

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Solved exercise

Example 1. *Solve by approximation of over* 0.25 *the equation over the reals*

$$\sqrt{x+1} = 2^x - 1$$

First of all, the existence set of the formula is

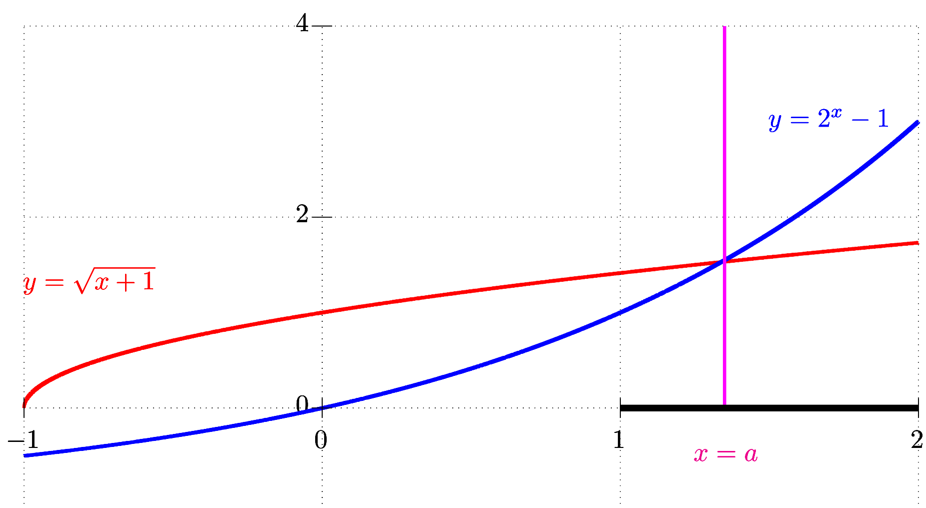
$$x \geq -1 \text{ for the existence of the root}$$

We use the graphical method - we determine the intersections points of the graph of the functions

$$F: \begin{array}{ccc} [-1,+\infty) & \longrightarrow & \mathbb{R} \\ x & \mapsto & \sqrt{x+1} \end{array} \quad G: \begin{array}{ccc} [-1,+\infty) & \longrightarrow & \mathbb{R} \\ x & \mapsto & 2^x - 1 \end{array}$$

The domain of both is the definition field of the formula, $[-1,+\infty)$, *since for* x*'s outside of it the equation has no meaning.*

GNU23: Solving $\sqrt{x+1} = 2^x - 1$



And $x = a$ *is the solution. We don't know the value of* a *precisely but we do know that* $1 < a < 2$*, and we see*

$$F(1) = \sqrt{2} > 1 = G(1) \text{ and } F(2) = \sqrt{3} < 3 = G(2)$$

We want to shrink that interval, and we take the middle point $x = \frac{1+2}{2} = 1.5$. *We have*

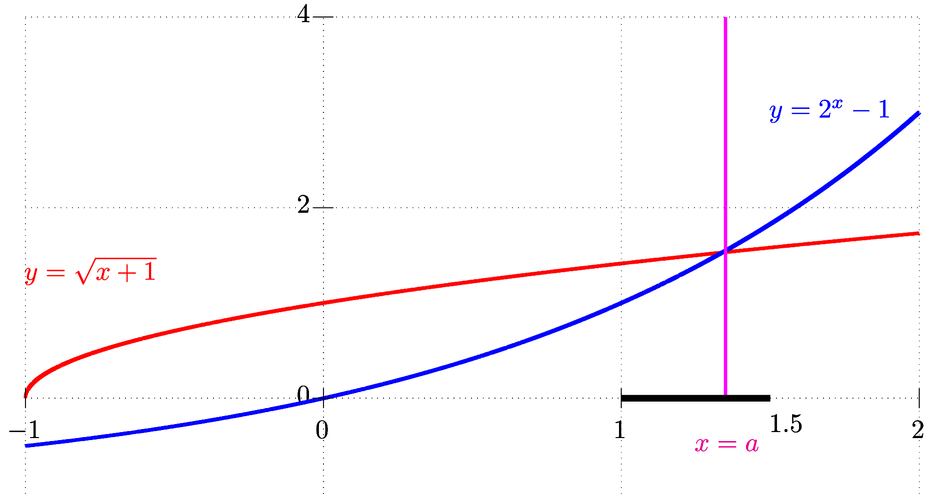
$$F(1.5) = \sqrt{2.5} \simeq 1.58 \text{ and } G(1.5) = 2^{3/2} - 1 = 2\sqrt{2} - 1 \simeq 1.82$$

so

$$F(1.5) < G(1.5)$$

and $1 < a < 1.5$

GNU24: Solving $\sqrt{x+1} = 2^x - 1$



We want to shrink this interval, and again we we take the middle point $x = \frac{1+1.5}{2} = 1.25$. *We have*

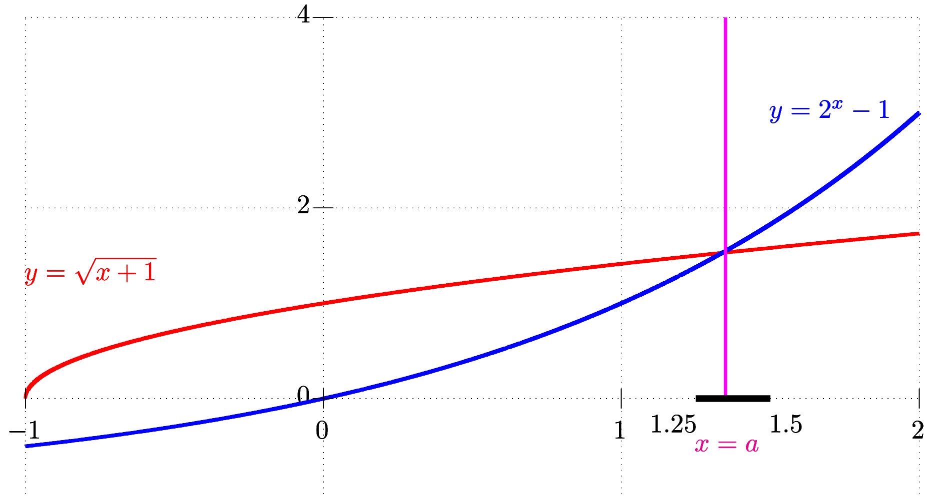
$$F(1.25) = \sqrt{2.25} \simeq 1.5 \text{ and } G(1.25) = 2^{5/4} - 1 = 2\sqrt[4]{2} - 1 \simeq 1.37$$

so

$$F(1.25) > G(1.25)$$

and $1.25 < a < 1.5$.

GNU25: Solving $\sqrt{x+1} = 2^x - 1$



The middle point of the interval is $\frac{1.25+1.5}{2} \simeq 1.37$*, while the length of the interval is* 0.25*, and so the approximation we have is good enough. If we want more precision, a better approximation, we continue the process of shrinking the interval.*

We now know that the solution is $x = a = 1.37 \pm 0.125$

Proposed exercises

The solutions may have more precision that requested. Prceed until you get the required precsion and no more.

Exercise 1. *Given the functions*

$$F: \begin{array}{ccc} \mathbb{R} & \longrightarrow & \mathbb{R} \\ x & \mapsto & 3^x \end{array} \quad \textit{and} \quad F: \begin{array}{ccc} \mathbb{R} & \longrightarrow & \mathbb{R} \\ x & \mapsto & -3x+10 \end{array}$$

find all the intersection of theirs graphs up to a precision of 0.1 *[Solution: one intersection in* $[1.53, 1.54]$*]*

Exercise 2. *Given the functions*

$$F: \begin{array}{ccc} \mathbb{R} & \longrightarrow & \mathbb{R} \\ x & \mapsto & \arctan(x+1) \end{array} \quad \textit{and} \quad F: \begin{array}{ccc} \mathbb{R} & \longrightarrow & \mathbb{R} \\ x & \mapsto & x^2 \end{array}$$

find all the intersection of theirs graphs up to a precision of 0.1 *[Solution: two intersections, one in* $[-0.615, -0.605]$ *and the other in* $[1.04, 1.06]$*]*

Exercise 3. *Given the functions*

$$F: \begin{array}{ccc} \mathbb{R} & \longrightarrow & \mathbb{R} \\ x & \mapsto & \sin(x) \end{array} \quad \textit{and} \quad F: \begin{array}{ccc} \mathbb{R} & \longrightarrow & \mathbb{R} \\ x & \mapsto & x^2 \end{array}$$

find all the intersection of theirs graphs up to a precision of 0.1 *[Solution: two intersections, one in* 0 *and the other in* $[0.8, 0.9]$*]*

Exercise 4. *Find the solutions of the equation* $x^3 - x^2 - 4 = 0$ *up to a precision of* 0.1 *[Hint: rewrite the equation as* $x^3 = x^2 + 4$ *and intersect the graphs of the functions*

$$F: \begin{array}{ccc} \mathbb{R} & \longrightarrow & \mathbb{R} \\ x & \mapsto & x^3 \end{array} \quad \textit{and} \quad F: \begin{array}{ccc} \mathbb{R} & \longrightarrow & \mathbb{R} \\ x & \mapsto & x^2+4 \end{array}$$

Solution: one intersection in $[1.99, 2.01]$*] Other Hint: look for a precise solution.*

Exercise 5. *Find the solutions of the equation* $x^3 - x^2 - 3 = 0$ *up to a precision of* 0.1 *[Hint: rewrite the equation as* $x^3 = x^2 + 3$ *and intersect the graphs of the functions*

$$F: \begin{array}{ccc} \mathbb{R} & \longrightarrow & \mathbb{R} \\ x & \mapsto & x^3 \end{array} \quad \textit{and} \quad F: \begin{array}{ccc} \mathbb{R} & \longrightarrow & \mathbb{R} \\ x & \mapsto & x^2+4 \end{array}$$

Solution: one intersection in $[1.85, 1.87]$*]*

Exercise 6 (Different)**.** *Find the value of sin*$(\sqrt{2})$ *up to a precision of* 0.1*. Hint: find a suitable small interval containing* $\sqrt{2}$ *first and the work with the* sin.