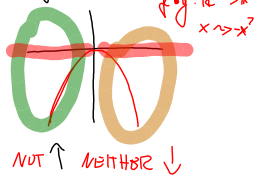
Ex

$f \uparrow \quad f(x) = x$

$g \downarrow \quad g(x) = -x$

$f \cdot g(x) = -x^2$

$f \cdot g : \mathbb{R} \to \mathbb{R}$, $x \rightsquigarrow -x^2$

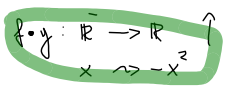


$\uparrow \cdot \downarrow$ = NOTHING

NOT $\uparrow$ NEITHER $\downarrow$

$f : \mathbb{R}^- \to \mathbb{R}$, $x \rightsquigarrow x$

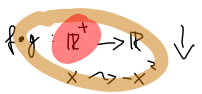
$g : \mathbb{R}^- \to \mathbb{R}$, $x \rightsquigarrow -x$



$f \cdot g : \mathbb{R}^- \to \mathbb{R}$, $x \rightsquigarrow -x^2$ $\uparrow$

$f : \mathbb{R}^+ \to \mathbb{R}$, $x \rightsquigarrow x$

$g : \mathbb{R}^+ \to \mathbb{R}$, $x \rightsquigarrow -x$



$f \cdot g : \mathbb{R}^+ \to \mathbb{R}$, $x \rightsquigarrow -x^2$ $\downarrow$

$f \uparrow \quad g \downarrow \overset{?}{\Longrightarrow} f \circ g \uparrow \quad (\forall f, g)$

IF WE WANT TO SHOW THAT $\Longrightarrow$ IS ALWAYS TRUE

WE NEED TO PROVE IT $x_1 < x_2 \Rightarrow f \circ g(x_1) < f \circ g(x_2)$

IF WE WANT TO SHOW THAT $\Longrightarrow$ IS SOMETIMES FALSE

WE NEED EXAMPLES

Ex $f(x) = \arctan(x^2)$

$f : \mathbb{R} \to \mathbb{R}$

$x \rightsquigarrow \text{ARCTAN}(x^2)$

E.F $\mathbb{R}$ $\mathbb{R}$

$f_1 : \mathbb{R} \to \mathbb{R}$, $x \to x^2$

$f_2 : \mathbb{R} \to \mathbb{R}$, $x \to \text{ARCTAN}(x)$

ZEROES. $\text{ARCTAN}(x^2) = 0$

$x = 0$

$\text{ARCTAN}(x^2) = 0$

$\text{TAN}(\text{ARCTAN}(x^2)) = \text{TAN}(0)$

$x^2 = 0$

$x = 0$

$f(x) > 0$ $\quad \text{ARCTAN}(x^2) > 0$

$\Uparrow$

$x^3 > 0 \Rightarrow x \neq 0$

IS f ODD, EVEN ?

$f(x) = \text{ARCTAN}(x^2)$

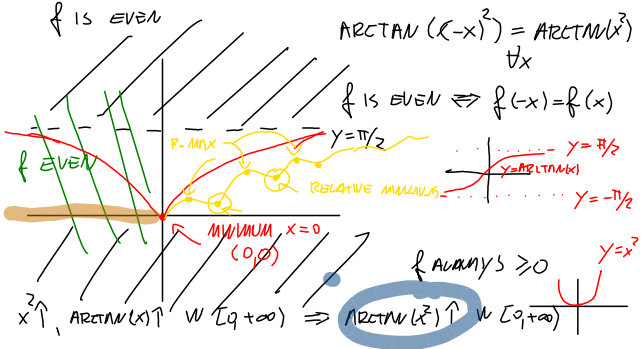
$=$

$\text{ARCTAN}((-x)^2) = \text{ARCTAN}(x^2)$

$\forall x$

f IS EVEN

f IS EVEN $\Leftrightarrow f(-x) = f(x)$



f ALWAYS ≥ 0

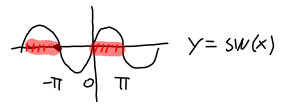
$\text{SIN}(x^2 - x) \geq 0$

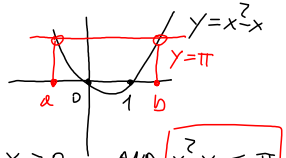
$\Uparrow$

$x^2 - x \in [0, \pi]$

$\in [-2\pi, -\pi]$

$x^2 - x \in [0, \pi] \Leftrightarrow$





$x^2 - x \geq 0$ $\quad$ AND $x^2 - x \leq \pi$

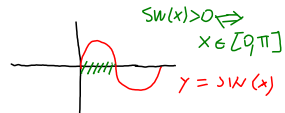
$\Uparrow$

$x \leq 0 \cup x \geq 1$

$\Downarrow$

AND GO ON

$\text{SIN}(x) > 0 \Leftrightarrow x \in [0, \pi]$



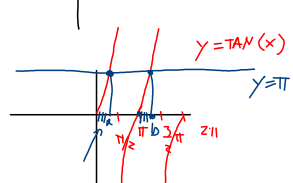
$\text{SIN}(\text{TAN}(x)) > 0$

$x \in [0, 2\pi)$

$\text{TAN}(x) \in [0, \pi]$

$\Uparrow$

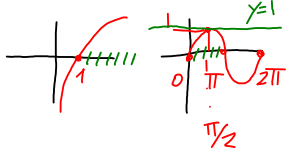
$0 \leq \text{TAN}(x) \leq \pi$



$[0, a] \cup [\pi, b]$

Ex $f(x) = \log(\sin(x))$

$x \in [0, 2\pi)$



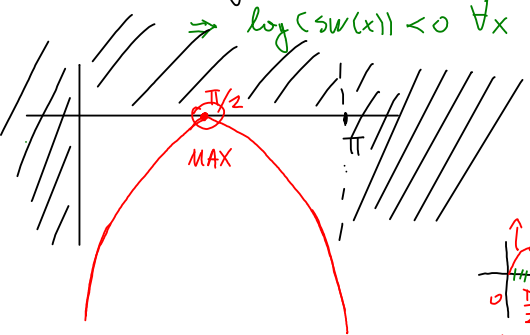
E.F. $\sin(x) > 0 \iff x \in (0, \pi)$

E.F: $(0, \pi)$

ZEROES: $\log(\sin(x)) = 0 \iff \sin(x) = 1 \iff x = \frac{\pi}{2}$

POSITIVITY: $\log(\sin(x)) > 0 \iff \sin(x) > 1$ ~~$\exists x$~~

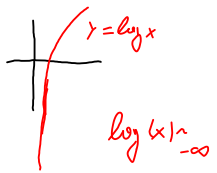
$\Rightarrow \log(\sin(x)) < 0 \ \forall x$



$\log(\sin(x)) \uparrow : x \in (0, \pi/2)$

$\log(\sin(x)) \downarrow : x \in (\pi/2, \pi)$

$(0, \pi)$ E.F

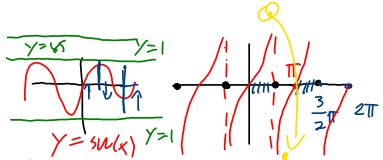


$\sin(x) \sim 0$

$\Updownarrow$

$x \sim 0$ or $x \sim \pi$

Ex $f(x) = TAN(SW(x))$



EF: $SW(x) \neq \pi/2 + K\pi \quad K \in \mathbb{Z}$

$\pi/2, -\pi/2, \frac{3}{2}\pi, -\frac{3}{2}\pi$

$SW(x) = \pi/2 \simeq 1.5$

EF: $\mathbb{R}$

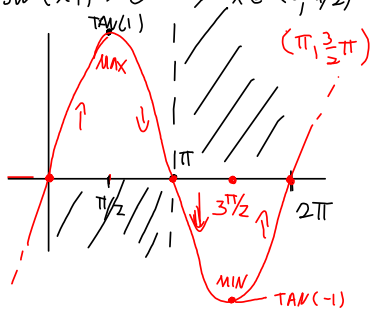
f IS ODD ∘ ODD = EVEN ⇒ WE ARE INTERESTED IN $\mathbb{R}^+$

f IS PERIODIC 2π IS A PERIOD OF f ⇒ WE ARE INTERESTED ONLY IN $[0, 2\pi)$

ZEROES

$TAN(SW(x)) = 0 \Rightarrow SW(x) = 0 \Rightarrow x = 0, x = \pi$

$TAN(SW(x)) > 0 \Rightarrow x \in (0, \pi/2)$



$f(0) = TAN(SW(0))$

$= TAN(0) = 0$

$f(2\pi) = TAN(SW(2\pi))$

$= TAN(0) = 0$

$f(\pi/2) = TAN(SW(\pi/2))$

$= TAN(1) \simeq 1.5$