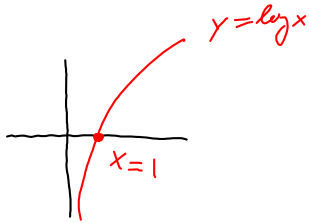
$f(x) = \log\sqrt{x}$ E.F. $x > 0$

$\sqrt{x}$ $x \geq 0$

$\log x$ $x > 0 \Rightarrow \sqrt{x} > 0$ $\Rightarrow$ $x > 0$

$f : (0, +\infty) \longrightarrow \mathbb{R}$

$x \rightsquigarrow \log\sqrt{x}$



ZEROES $\log\sqrt{x} = 0$

$x = 1$

$\Uparrow$

$\sqrt{x} = 1 \iff x = 1$

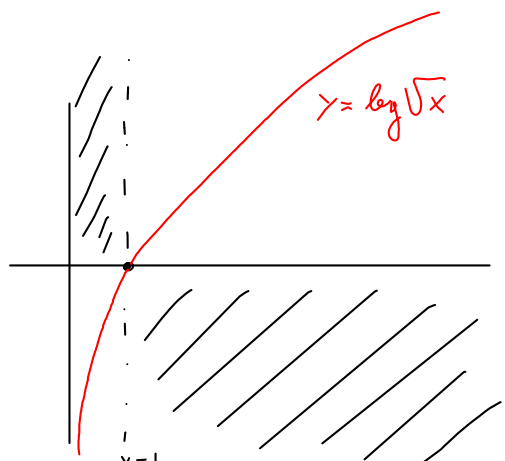
POSITIVITY $\log\sqrt{x} > 0$

$x \geq 1$

$\sqrt{x} \geq 1 \iff x \geq 1$

apply $y = x^2$ ↑ in $x \geq 0$

f is MONOTONE ? YES f is ↑



$f : (0, +\infty) \longrightarrow \mathbb{R}$

$x \rightsquigarrow |\log \sqrt{x}|$

$g : (0, +\infty) \longrightarrow \mathbb{R}$

$x \rightsquigarrow -\log \sqrt{x}$

$h : \mathbb{R} - \{0\} \longrightarrow \mathbb{R}$

$x \rightsquigarrow \log \sqrt{|x|}$

$T : (-\infty, 0) \longrightarrow \mathbb{R}$

$x \rightsquigarrow \log \sqrt{-x}$

$f : \mathbb{R} \to \mathbb{R}$

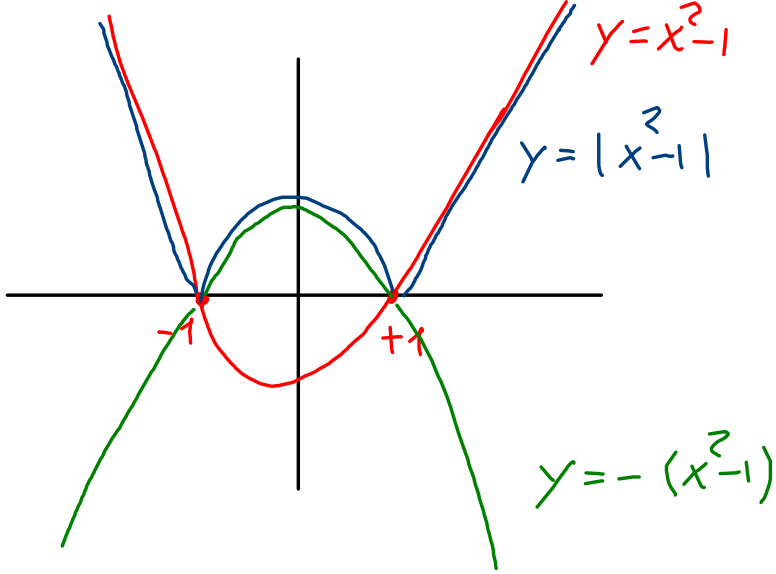
$x \rightsquigarrow x^2 - 1$

$g : \mathbb{R} \to \mathbb{R}$

$x \rightsquigarrow |x^2 - 1|$

$h : \mathbb{R} \to \mathbb{R}$

$x \rightsquigarrow -(x^2 - 1)$



$T : \mathbb{R} \to \mathbb{R}$

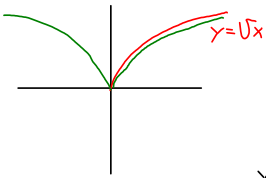
$x \rightsquigarrow |x|^2 - 1$

$T = f$

$f: \mathbb{R} \to \mathbb{R}$ FUNCTION WE KNOW GRAPH (f)

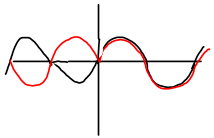
$|f|$ GRAPH: THE SAME WHEN $f \geq 0$
THE OPPOSITE WHEN $f < 0$

$f(|x|)$ GRAPH: THE SAME WHEN $x \geq 0$
$f(-x)$ WHEN $x < 0$

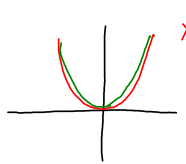


$y = \sqrt{|x|}$ | WE SYMMETRIZE THE GRAPH OF f W.R.T. $x=0$

$y = \sin(x)$ $y = \sin(|x|)$

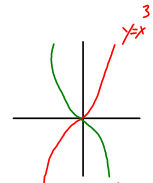


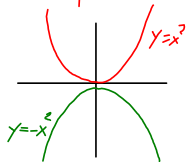
$y=(-x)^3$



$f(-x)$

$y=(-x)^2$



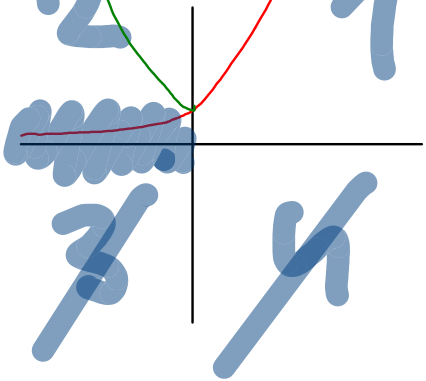


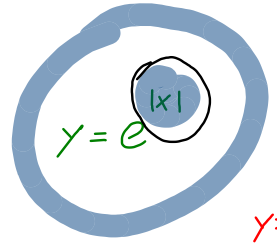
$-f(x)$: THE OPPOSITE OF $f(x)$

$f(x) = e^x$

$y = e^x$

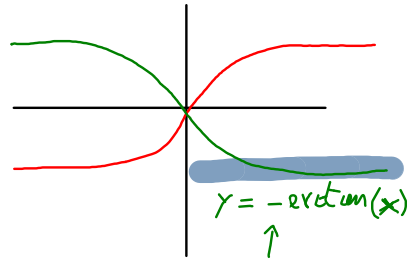
2 1 3 4





$$e^{|x|} = \begin{cases} e^x & x \geq 0 \\ e^{-x} & x < 0 \end{cases}$$

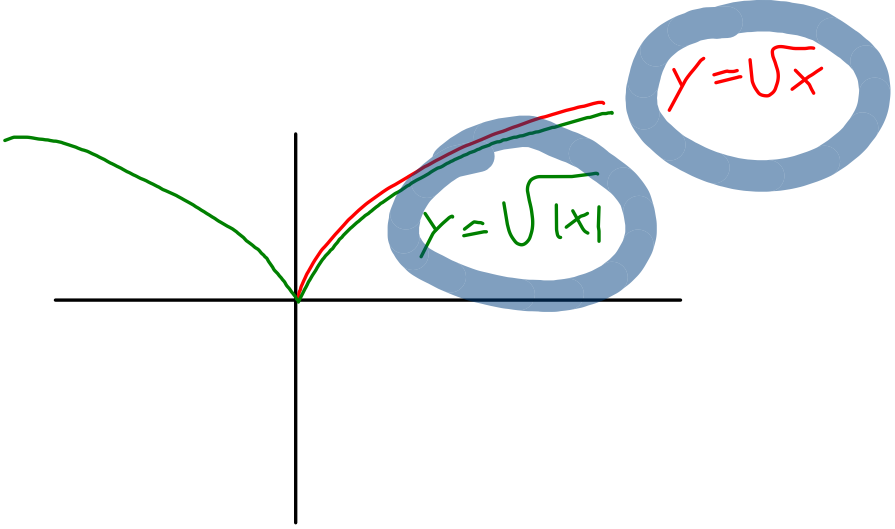
$y = \text{evencon}(x)$



$e^{-x} = e^x$

$x < 0 \qquad x \geq 0$

$e^{|-5|} = e^5$



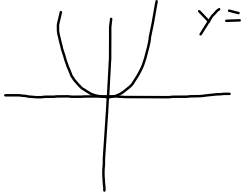
$f : [0,+\infty) \to \mathbb{R}$

$x \rightsquigarrow \sqrt{x}$

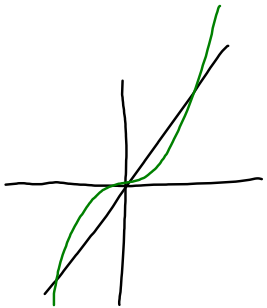
$y : \mathbb{R} \to \mathbb{R}$

$x \rightsquigarrow \sqrt{|x|}$

$\sqrt{|-1|} = \sqrt{1} \in \mathbb{R}$



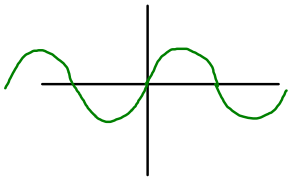
symmetric w.r.t. $x=0$



$y=x$

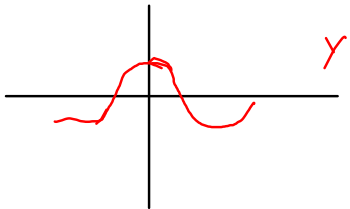
$y=x^3$

symmetric w.r.t the point $(0,0)$



$y=\sin(x)$

sy. w.r.t. $(0,0)$



$y=\cos(x)$

sy. w.r.t $x=0$

A FUNCTION F IS CALLED

EVEN IF ITS GRAPH IS SY. W.R.T $x = 0$

ODD IF " " " W.R.T. $(0,0)$

F IS EVEN $\iff f(x) = f(-x)$ EQUIV

F IS ODD $\iff f(x) = -f(-x)$ $\left[f(-x) = -f(x) \right]$

$\forall x \in \mathbb{R}$

EVEN : $x^2, x^4, x^{2n}, \cos(x), |x|, K$ CONSTANT

ODD : $x, x^3, x^{2n+1}, \sin(x), \tan(x), \arctan(x)$

NOR EVEN NOR ODD $e^x, \sqrt{x}, \log, x^2+2x+1$ (sy $x=0$)

$f : \mathbb{R} \to \mathbb{R}$ IS ODD, EVEN ?
$x \mapsto 3$

$\forall x \; f(x) \overset{?}{=} f(-x)$ YES
$3 \quad 3$

f sy w.r.t. $(0,0)$

g sy w.r.t. $(0,0)$

$\Downarrow$

$f+g$ IS SY W.R.T. $(0,0)$

EVEN + EVEN = EVEN

ODD + ODD = ODD

ALG : $(f+g)(-x) \overset{?}{=} -(f+g)(x)$ YES

$f(-x) + g(-x) = -f(x) - g(x) = -(f+g)(x)$

f, g ODD

$= -(f+g)(x)$

EVEN + ODD $\overset{?}{=}$.

$x^2 + \sin(x)$

EVEN ODD

[NOR ODD NOR EVEN]

$\forall x \; (-x)^2 + \sin(-x) \overset{?}{=} -x^2 - \sin(x)$

$x^2 - \sin(x) \overset{?}{=} -x^2 - \sin(x)$ NO

$\Downarrow$ $\quad << 0$

f ODD g ODD $\quad f \cdot g$?

$$x^3 \cdot x^5 = x^8$$

ODD ODD EVEN

ODD · ODD = EVEN WE ARE TRYING TO PROVE THAT

$(f \cdot g)(x)$

$$\forall x \quad f(x) \cdot g(x) \overset{?}{=} f(-x) \cdot g(-x) = (f \cdot g)(-x)$$

YES

f, g ODD

$$-f(-x) \cdot -g(-x) = f(-x) \cdot g(-x)$$

ODD · EVEN = ?

$\forall x$

$f(-x) \cdot g(-x) = f(-x) \cdot g(x)$ — g is EVEN, $g(x) = g(-x)$

$= -f(x) \cdot g(x)$ — f is ODD, $f(-x) = -f(x)$

$= -(f(x) \cdot g(x))$

$\Rightarrow$ ODD · EVEN = EVEN

EVEN ∘ EVEN

$\cos(x^2)$

$\arctan(x^3)$

ODD ∘ ODD

$\tg(x^3+x)$

ODD ∘ ODD

x^3+x

ODD + ODD = ODD

$f(|x|)$ EVEN ALWAYS ($\forall f$)

$f(x^2)$ " "

$f \circ g$ IS EVEN

ODD ∘ EVEN $\overset{?}{=}$ EVEN $\quad \forall x \quad f\circ g(-x) \overset{?}{=} f\circ g(x)$

$f(-x) = -f(x)$ f ODD

$g(-x) = g(x)$ g EVEN

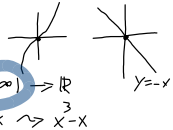
$f(g(-x))$

" g EVEN

$f(g(x))$ = " OK

$f(x) = x^3 - x$ $\quad f: \mathbb{R} \to \mathbb{R}$ $\quad E.F = \mathbb{R}$

$x^3, -x$ ARE ODD



f IS ODD $\Rightarrow$ WE CAN STUDY $f: [0,+\infty) \to \mathbb{R}$, $x \rightsquigarrow x^3 - x$

ZEROES : $f(x)=0 \iff x^3 - x = 0 \quad x(x^2-1) = 0$

$0, 1, \cancel{-1}$

$f(x) \geq 0 \iff x^3 - x \geq 0 \iff x(x+1)(x-1) \geq 0$

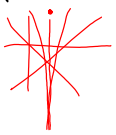
$x \geq 1$

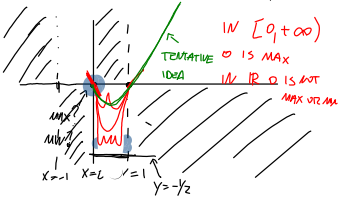
0 0

W $[0,+\infty)$

IN $[0,+\infty)$ $x^3 - x \geq 0$ IF $x-1 \geq 0 \iff x \geq 1$

$f(10) = 1000 - 10$





$f(1/2) = (+1/2)^3 - 1/2 = -1/8 - 1/2$

$-1/2$

$f(+3/4) = \frac{3^3}{4^3} - \frac{1}{2}$

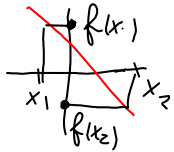
$= \frac{27}{64} - \frac{1}{2}$

$x^3 - x$ $\quad x \in (0,1)$

POSITIVE $\quad$ NEGATIVE $\quad$ IS NEGATIVE

$\geq -1/2$

$f \downarrow \quad g \downarrow \qquad f \circ g \downarrow \; ?$



$x_1 < x_2$

FLIPS

$g(x_1) > g(x_2)$ $\qquad g \downarrow \qquad x_1 < x_2$

$f(g(x_1)) < f(g(x_2)) \quad f \downarrow \qquad f(x_1) > f(x_2)$

FLIPS

$x_1 < x_2 \qquad f \circ g\,(x_1) < f \circ g\,(x_2)$

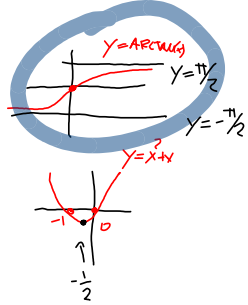
$\Downarrow \quad f \circ g \uparrow$

MULTIPLYW BY -1 $\iff$ AS APPLYWG $\qquad f : \mathbb{R} \to \mathbb{R} \downarrow$

$x \rightsquigarrow -x$

$f \downarrow \quad g \uparrow \qquad f \circ g \; ?$

$f(x) = \text{ARCTAN}(x^2+x)$ EF : $\mathbb{R}$



ZEROES :

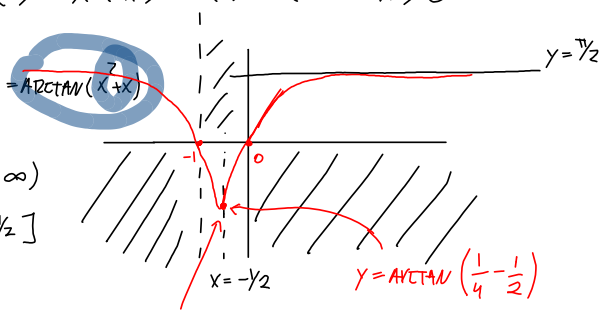
$\text{ARCTAN}(x^2+x) = 0 \iff x^2+x=0$

$x = 0, -1$

F NOR ODD NOR EVEN

$f(x) \geq 0 \iff x^2+x \geq 0 \iff x \leq -1 \cup x \geq 0$

$f \uparrow ?$ $f(x) = \text{ARCTAN}(x^2+x)$



$x^2+x \uparrow \; [-1/2, +\infty)$

$x^2+x \downarrow \; (-\infty, -1/2]$

$\text{ARCTAN}(x) \uparrow$

MIN

$y = \text{ARCTAN}\left(\frac{1}{4} - \frac{1}{2}\right)$

$= \text{ARCTAN}(-1/2)$

$= -\text{ARCTAN}(1/2)$

$f(|x|)$ $f(g(x))$

g is sym w.r.t. $x=a \Rightarrow f \circ g(x)$ is sym w.r.t $x=a$

$f \uparrow \quad g \uparrow$ $\quad f \circ g \uparrow \quad f+g \uparrow \quad f-g \; ? \quad f \cdot g \uparrow \quad \frac{f}{g} \; ?$

$f \uparrow \quad g \downarrow$

$\uparrow \text{ARCTAN}\left(\underbrace{x^2 \downarrow + x}\right)$

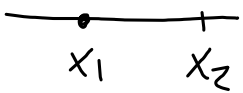
$W \; (-\infty, -1/2]$

$x_1 < x_2$

$\Downarrow$

$g(x_1) > g(x_2)$

$g \downarrow$



$f(g(x_1)) > f(g(x_2))$

$f \uparrow$

$f \circ g(x_1) > f \circ g(x_2)$

$f \circ g \downarrow$

$f \circ g(x_1) < f \circ g(x_2)$