FCS
Math: Functions

Massimo Caboara

May 13^{th}, 2021

Definition 1. *A function $F : \mathbb{R} \longrightarrow \mathbb{R}$, is*

- ODD *if $\forall\ x \in A\ F(-x) = -F(x)$ or, equivalently, the graph of F is symmetric with respect to the origin.*
- EVEN *if $\forall\ x \in A\ F(-x) = F(x)$ or, equivalently, the graph of F is symmetric with respect to the $x = 0$ vertical line.*

Example 1. *The functions with the following formulas are ODD.*

$$f(x) = 0,\ f(x) = x,\ f(x) = x^3,\ f(x) = x^{2n+1}, n \in \mathbb{N},\ y = \sin(x), y = \tan(x),\ y = \arcsin(x),\ y = \arctan(x)$$

Example 2. *The functions with the following formulas are EVEN.*

$$f(x) = a,\ a \in \mathbb{R},\ \ f(x) = x^2,\ f(x) = x^{2n}, n \in \mathbb{N},\ f(x) = \cos(x),\ f(x) = \arccos(x),\ f(x) = \mid x \mid$$

Example 3. *The functions with the following formulas are neither ODD nor EVEN.*

$$f(x) = x + 1,\ f(x) = \sqrt{x},\ f(x) = e^x,\ y = \log(x),\ y = x^2 + x$$

Proposition 1. *If the invertible function $F : \mathbb{R} \longrightarrow \mathbb{R}$ is ODD (EVEN), then $F^{-1} : B \longrightarrow A$ is ODD (EVEN).*

Proposition 2. *If the invertible, continuous function $F : \mathbb{R} \longrightarrow \mathbb{R}$ is increasing (decreasing), then $F^{-1} : B \longrightarrow A$ is increasing (decreasing).*

Proposition 3. *We have the functions f, g*

f *odd*	g *odd*	$f \pm g$ *are odd*	$f \cdot g, f/g$ *are even*	$f \circ g$ *is odd*
f *even*	g *even*	$f \pm g$ *are even*	$f \cdot g, f/g$ *are even*	$f \circ g$ *is even*
f *odd*	g *even*	$f \pm g$ *neither*	$f \cdot g, f/g$ *are odd*	$f \circ g$ *is even*
f *even*	g *odd*	$f \pm g$ *neither*	$f \cdot g, f/g$ *are odd*	$f \circ g$ *is even*

Note that if g is EVEN, $f \circ g$ is EVEN for any function.

Proposition 4. *We have the monotone, continuous functions f, g defined on an interval (a, b)*

$$\begin{array}{ll} f\uparrow & -f\downarrow \\ f\downarrow & -f\uparrow \\ f\uparrow & 1/f\downarrow \\ f\downarrow & 1/f\uparrow \end{array}$$

$$\begin{array}{lllll} f\uparrow & g\uparrow & f+g\uparrow & f\cdot g\ ? & f\circ g\uparrow \\ f\downarrow & g\downarrow & f+g\downarrow & f\cdot g\ ? & f\circ g\uparrow \\ f\uparrow & g\downarrow & f+g\ ? & f\cdot g\ ? & f\circ g\ \downarrow \\ f\downarrow & g\uparrow & f+g\ ? & f\cdot g\ ? & f\circ g\ \downarrow \end{array}$$