Ex I

$\mathbb{N} = \{0, 1, 2, \dots\}$

"DIVIDES"

$A = \{a \in \mathbb{N} \mid a \text{ is a MULTIPLE OF } 12\}$

$= \{a \in \mathbb{N} \mid 12 \text{ DIVIDES } a\} = \{a \in \mathbb{N} \mid 12 \mid a\}$

$= \{12, 24, 36, 48, 60, \dots\}$

$\Leftrightarrow \exists k \in \mathbb{N} \text{ s.t. } a = 12k$

$A = \{0, 12, 24, 36 \dots\}$

$k = 1 \quad 12 \in A$

0 IS DIVIDED BY ANY NATURAL NUMBER $k = 2 \quad 24 \in A$

$A = \{12k \mid k \in \mathbb{N}\} = \{\cancel{0}, 12, 24, 36, 48, \cancel{60}, 72, \dots\}$

$B = \{15a \mid a \in \mathbb{N}\} = \{0, 15, 30, 45, 60, 75, \dots\}$

$A \cap B = \{60k \mid k \in \mathbb{N}\}$

$\text{LCM}(12, 15) = 60$

$A \cup B = \{0, 12, 15, 24, 30, 36, 45, 48, 60, \dots\}$ ←

$A - B = \{12, 24, 36, 48, 72, 84, \dots\}$ ← NO 60, NO 0

60+12 60+24

$= \{12 + 60k, 24 + 60k, 36 + 60k, 48 + 60k \mid k \in \mathbb{N}\}$

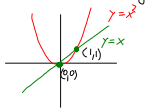
Ex 3 $A = \{(a, a^2) \mid a \in \mathbb{R}\}$ $B = \{(b, b) \mid b \in \mathbb{R}\}$

$A, B \subseteq \mathbb{R} \times \mathbb{R} = \mathbb{R}^2$

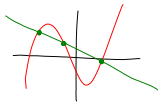
I METHOD: $f : \mathbb{R} \to \mathbb{R}$, $a \rightsquigarrow a^2$ $\quad g : \mathbb{R} \to \mathbb{R}$, $b \rightsquigarrow b$

$A = GR(f)$ $\quad B = GR(g)$

$A = \{(a, f(a)) \mid a \in \mathbb{R}\}$



$A \cap B = \{(0,0), (1,1)\}$



II METHOD $\quad A = \{(a, a^2) \mid a \in \mathbb{R}\} \subseteq \mathbb{R}^2$

$A = \{(a, f(a)) \mid a \in \mathbb{R}\}$ $\quad f : \mathbb{R} \to \mathbb{R}$, $x \rightsquigarrow x^2$

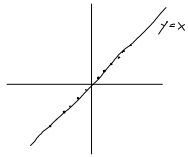
$A = \{(x, y) \in \mathbb{R}^2 \mid y = x^2\}$ $\quad (\pi, \pi) \dots$

PARAMETER

PARAMETRIC: $B = \{(b, b) \mid b \in \mathbb{R}\} = \{(1,1), (2,2), (\tfrac{1}{2}, \tfrac{1}{2}), \dots\}$

CARTESIAN EQUATION

CARTESIAN: $B = \{(x, y) \in \mathbb{R}^2 \mid y = x\}$



$A \cap B = \{(x, y) \in \mathbb{R}^2 \mid (x, y) \in A \text{ AND } (x, y) \in B\}$

$= \{(x, y) \in \mathbb{R}^2 \mid y = x^2 \text{ AND } y = x\}$

$= \{(x, y) \in \mathbb{R}^2 \mid \begin{cases} y = x^2 \\ y = x \end{cases}\} = \{(1,1), (0,0)\}$

$\downarrow$

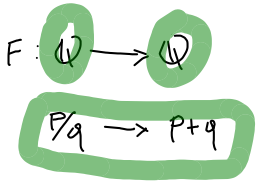
$x = x^2 \iff x^2 - x = 0 \iff x(x-1) = 0$

$x = 1, 0$ $\quad x = 1 \Rightarrow y = 1$

$x = 0 \Rightarrow y = 0$

Ex 10

QUESTION: IS THIS A FUNCTION?



$\frac{1}{1} = \frac{2}{2} = \frac{9}{9}$

$F(1) = F(\frac{1}{1}) = 1+1 = 2$

$F(\frac{3}{2}) = 3+2 = 5$ ✗

$1 = \frac{3}{2}$ $\quad F(\frac{3}{2}) = 4$

$G : \mathbb{N} \to \mathbb{N}$

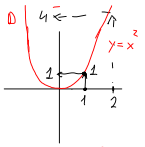
$a+b-b \rightsquigarrow a^2+b^2$

$3+0+0 = 3+1-1$ $\quad G(3+1-1) = 3^2+1^2 = 10$

$G(3+0-0) = 3^2+0^2 = 9$

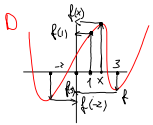
$\mathbb{R}^2 \supseteq D$ SET D

WHEN D IS THE GRAPH OF A FUNCTION ?



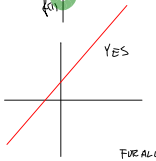
$$D = \{(x, x^2) \in \mathbb{R}^2 \mid x \in \mathbb{R}\} = \{(x, f(x)) \mid x \in \mathbb{R}\}$$

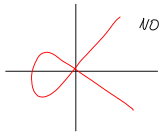
$$f : \mathbb{R} \to \mathbb{R}, \quad x \rightsquigarrow x^2 = f(x)$$



$f : \mathbb{R} \to \mathbb{R}$, $x \rightsquigarrow f(x)$

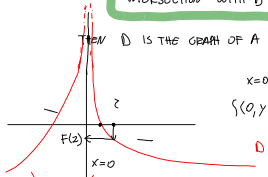
$F : \mathbb{R} \to \mathbb{R}$, $1 \rightsquigarrow ?$





$D \subseteq \mathbb{R}^2$ s.t. FOR ALL $\forall$ VERTICAL LINES THERE IS ONLY ONE INTERSECTION WITH D

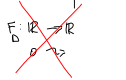
THEN D IS THE GRAPH OF A FUNCTION



$x = 0 \cap D = \emptyset$

$\{(0, y) \mid y \in \mathbb{R}\} \cap D = \emptyset$

$F_D(1) = 0$ F $G : \mathbb{R} - \{0\} \to \mathbb{R}$, $x \rightsquigarrow G(x)$



A, B sets $\quad D \subseteq A \times B$

D IS THE GRAPH OF A FUNCTION $F : A \to B$

$\Leftrightarrow \forall a \in A \ \exists! \ b \in B$ s.t. $(a, b) \in D$ $\quad F(a) = b$

$\exists!$: EXISTS ONLY ONE

$f: \mathbb{R} \to \mathbb{R}$, $x \rightsquigarrow \frac{1}{x^2}$

NO FUNCTION

$f: \mathbb{R}\text{-}\{0\} \to \mathbb{R}$, $x \rightsquigarrow \frac{1}{x^2}$

FUNCTION

$\nexists f(0)$

DOES NOT EXISTS

FORMULA : $\frac{1}{\sqrt{x^2-1}}$

WHERE DOES IT EXISTS ?

$\neq 0$ $\quad \sqrt{x^2-1} \neq 0$

≥ 0 $\quad x^2-1 \geq 0$

$y = x^2-1$

$\Updownarrow$

$x^2-1 > 0$

$\Updownarrow$

$x < -1$ AND $x > 1$

MAYBE IS A FUNCTION

MAYBE NOT

DEF

$f: \mathbb{R} \to \mathbb{R}$

$x \rightsquigarrow f(x)$

E.F(f) $= \{x \in \mathbb{R} \mid f(x) \text{ HAS SENSE}\}$

$f': \text{E.F}(f) \to \mathbb{R}$ IS A FUNCTION

$x \rightsquigarrow f(x)$

$f : \mathbb{R} \to \mathbb{R}$

$x \rightsquigarrow \frac{1}{x}$

NOT A FUNC.

$E.F.(f) = \mathbb{R} - \{0\}$

$f' : \mathbb{R} - \{0\} \to \mathbb{R}$

$x \rightsquigarrow \frac{1}{x}$

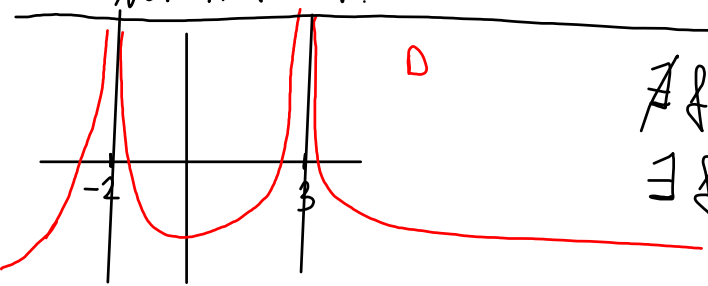
$f : \mathbb{R} \to \mathbb{R}$

$x \rightsquigarrow \frac{1}{(x-1)(x+1)}$

$f' : \mathbb{R} - \{\pm 1\} \to \mathbb{R}$

$x \rightsquigarrow \frac{1}{(x-1)(x+1)}$

NOT A FUNCT.



D

IS A FUNCTION

$\nexists f : \mathbb{R} \to \mathbb{R}$ S.T. $GR(f) = D$

$\exists f' : \mathbb{R} - \{-2, 3\} \to \mathbb{R}$

S.T. $GR(f') = D$

$f : \mathbb{R} \to \mathbb{R}$

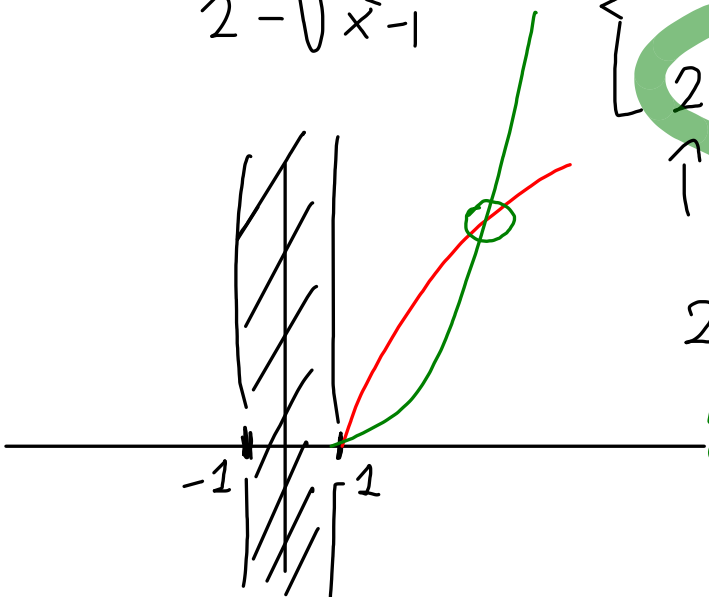
WHAT IS THE E.F. (f)

$$x \rightsquigarrow \frac{1}{2^x - \sqrt{x^2-1}}$$

$$\begin{cases} x^2 - 1 \geq 0 \iff x \leq -1 \cup x \geq 1 \\ 2^x - \sqrt{x^2-1} \neq 0 \end{cases}$$

$$2^x = \sqrt{x^2-1}$$

g f'



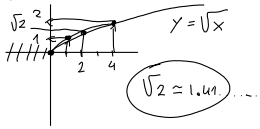
WE KNOW : PARABOLAS $y = ax^2 + bx + c$

LINES $y = x$

WHAT OTHER FUNCTIONS ?

$y = \sqrt{x}$

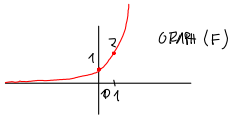
$x \geq 0$



$f : \mathbb{R}_0^+ \longrightarrow \mathbb{R}$

$\uparrow$

$\{ x \in \mathbb{R} \mid x \geq 0 \}$



GRAPH (F)

$F : \mathbb{R} \to \mathbb{R}$

$x \rightsquigarrow 2^x$

$F(0) = 2^0 = 1$

$F(1) = 2^1 = 2$

$F(2) = 4$

$F(3) = 2^3 = 8$

$F(-1) = 2^{-1} = 1/2$

$F(-2) = 1/4$

$G : \mathbb{R} \to \mathbb{R}$

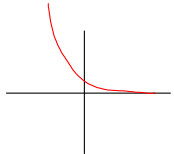
$x \rightsquigarrow 3^x$

5^x

10^x

$H : \mathbb{R} \to \mathbb{R}$

$x \rightsquigarrow (1/2)^x$

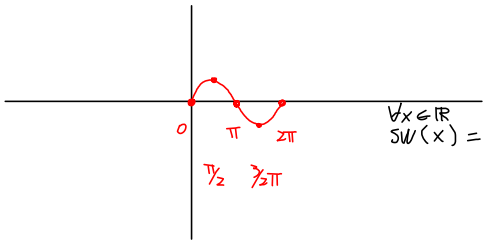


$F : \mathbb{R} \to \mathbb{R}$

$x \rightsquigarrow sw(x)$

$sw(0) = 0$
$sw(\pi) = 0$
$sw(2\pi) = 0$

$sw(\pi/2) = 1$
$sw(3/2\pi) = -1$
$sw(\pi/4)$
$sw(\pi/3)$
$sw(\pi/6)$



$\forall x \in \mathbb{R}$
$sw(x) = sw(x + 2\pi)$

$F : \mathbb{R} \to \mathbb{R}$

$p \in \mathbb{R}$ IS A PERIOD IF AND
$\forall x \in \mathbb{R} \quad F(x) = F(x+p)$

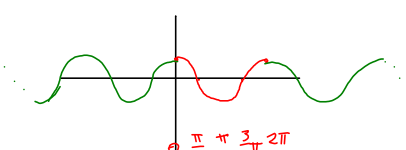
FOR $F : \mathbb{R} \to \mathbb{R}$
$x \rightsquigarrow sw(x)$

2π, 4π, 6π ARE PERIODS

$p \in \mathbb{R}$ IS THE PERIOD IF
$p = \min \{ q \in \mathbb{R} \mid q \text{ IS A PERIOD OF } F \}$
$q > 0$

$F : \mathbb{R} \to \mathbb{R}$

$x \rightsquigarrow \cos(x)$



$H : \mathbb{R} \to \mathbb{R}$

$x \rightsquigarrow \tan(x)$

$H' : \mathbb{R} - \left\{ k\frac{\pi}{2} \right\}_{k \in \mathbb{N}} \to \mathbb{R}$

$x \rightsquigarrow \tan(x)$

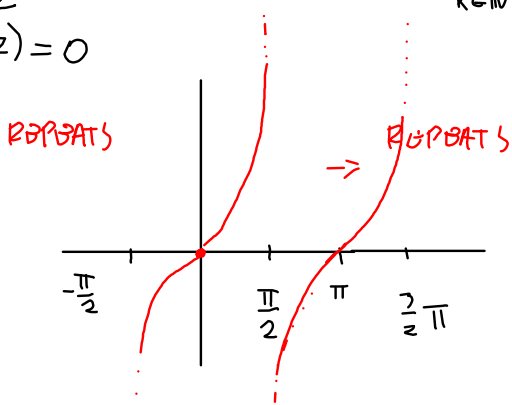
IS A FUNCTION

$\tan\left(\frac{\pi}{2}\right) = ?$

"

$\frac{\sin(\pi/2)}{\cos(\pi/2)} = 0$

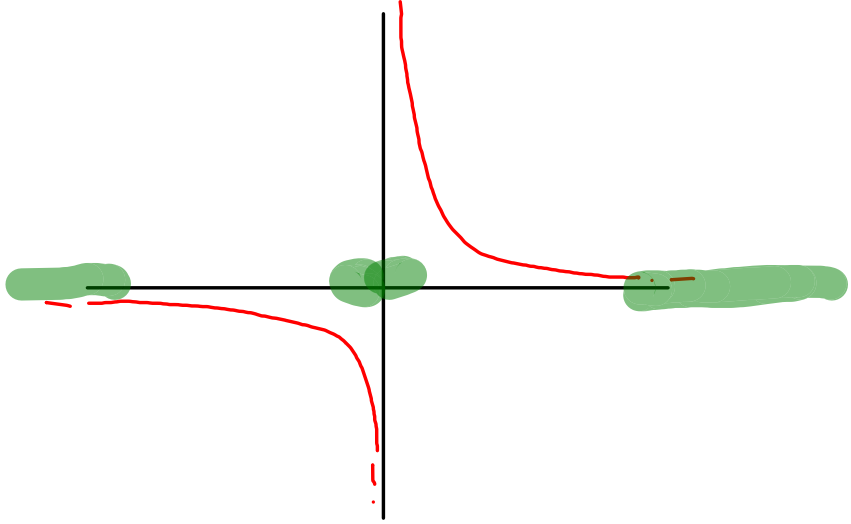
$D.F(H) = \mathbb{R} - \left\{ k\frac{\pi}{2} \right\}_{k \in \mathbb{N}}$



PERIOD = π

$F : \mathbb{R} - \{0\} \to \mathbb{R}$

$x \rightsquigarrow 1/x$



$y = x^2 - 3$

$y = 3x + 2$