FCS
Math: Functions
Definitions

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Definition 1. *Given* A, B *sets, the set* $D \subset A \times B$ *is the set of a function*

$$F_D : A \longrightarrow B$$

if and only if

$$\forall\ a \in A\ \exists!\ b \in B \quad s.t.\ (a, b) \in D$$

And the associated *function is*

$$\begin{array}{rccc} F_D : & A & \longrightarrow & B \\ & a & \mapsto & F_D(a) = b \end{array}$$

Corollary 1. *For a subsets* $D \subset \mathbb{R}^2$ *to be the graph of a function* $f : \mathbb{R} \longrightarrow \mathbb{R}$*, a vertical line has to intersect* D *once and only once.*

Corollary 2. *For a subsets* $D \subset \mathbb{R}^2$ *to be the graph of a function* $f : A \longrightarrow \mathbb{R}$*,* $A \subseteq \mathbb{R}$ *a vertical line has to intersect* D *at most once.*

Definition 2. *Given a "quasi function"*

$$\begin{array}{rccc} \mathcal{F} : & A & \longrightarrow & B \\ & a & \mapsto & \mathcal{F}(a) \end{array}$$

the existence field *of* $\mathcal{F}$*, or* $EF(\mathcal{F})$ *is the set*

$$a \in A \quad s.t.\ \textit{the formula } \mathcal{F}(a) \textit{ is meaningful.}$$

The associated *function to* $\mathcal{F}$ *is*

$$\begin{array}{rccc} F_{\mathcal{F}} : & EF(\mathcal{F}) & \longrightarrow & B \\ & a & \mapsto & \mathcal{F}(a) \end{array}$$

Definition 3. *Given a function*

$$\begin{array}{rccc} F : & \mathbb{R} & \longrightarrow & \mathbb{R} \\ & x & \mapsto & F(x) \end{array}$$

- *A* period *of F is an $p \in \mathbb{R}^+$ s.t.*

  $$\forall\ x \in \mathbb{R}\ f(x+p) = f(x)$$

  if any such p exists. N.B. A period is always strictly positive.

- *THE period of F is the minumum of all the periods, if such minumum exists.*

  $$Period(F) = \min\{p \in \mathbb{R}\ \mid\ p \text{ is a period of } F\}$$