

FCS
Math: Functions
Exercises

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Exercise 1. *Determine the existence fields for the the following "quasi functions" and write down explicitly the associated functions.*

1.

$$\begin{aligned} F: \mathbb{R} &\longrightarrow \mathbb{R} \\ x &\mapsto \sqrt{\frac{1}{x}} \end{aligned}$$

$$[EF(F) = (0, +\infty)]$$

2.

$$\begin{aligned} F: \mathbb{R} &\longrightarrow \mathbb{R} \\ x &\mapsto \sqrt{\frac{x-1}{x}} \end{aligned}$$

$$[EF(F) = (-\infty, 0) \cup [1, +\infty)]$$

3.

$$\begin{aligned} F: [0, 2\pi) &\longrightarrow \mathbb{R} \\ x &\mapsto \sqrt{\sin(x)} \end{aligned}$$

$$[EF(F) = [0, \pi]]$$

4.

$$\begin{aligned} F: \mathbb{R} &\longrightarrow \mathbb{R} \\ x &\mapsto \sqrt{\sin(x)} \end{aligned}$$

$$[EF(F) = \bigcup_{k \in \mathbb{Z}} [2k\pi, \pi + 2k\pi]]$$

5.

$$\begin{aligned} F: \mathbb{R} &\longrightarrow \mathbb{R} \\ x &\mapsto \sqrt{\sin(x) \cos(x)} \end{aligned}$$

$$[EF(F) = \bigcup_{k \in \mathbb{Z}} ([2k\pi, \pi/2 + 2k\pi] \cup [\pi + 2k\pi, 3\pi/2 + 2k\pi])]$$

6.

$$F: \mathbb{R} \longrightarrow \mathbb{R} \\ x \mapsto \sqrt{\sin(x) - 1}$$

$$[EF(F) = \{\pi/2 + 2k\pi \mid k \in \mathbb{Z}\}]$$

7.

$$F: \mathbb{R} \longrightarrow \mathbb{R} \\ x \mapsto \frac{x^2 - 1}{(x+1)^2}$$

$$EF(F) = \{x \in \mathbb{R} \mid x \neq -1\}$$

8.

$$F: \mathbb{R} \longrightarrow \mathbb{R} \\ x \mapsto \frac{\sqrt{x^2 - 2x}}{x^2 - 3x}$$

$$EF(F) = (-\infty, 0) \cup [2, 3) \cup (3, +\infty)$$

9.

$$F: \mathbb{R} \longrightarrow \mathbb{R} \\ x \mapsto \log(|x|)$$

$$EF(F) = (-\infty, 0) \cup (0, +\infty)$$

10.

$$F: \mathbb{R} \longrightarrow \mathbb{R} \\ x \mapsto \log(\sin(x))$$

$$[EF(F) = \bigcup_{k \in \mathbb{Z}} (2k\pi, \pi + 2k\pi)]$$

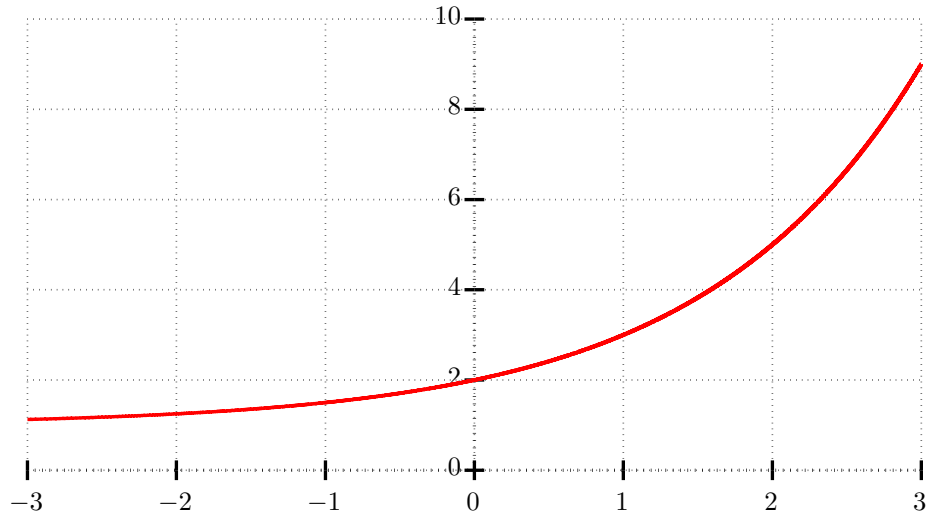
Exercise 2. After determining the existence field of the "quasi functions", draw the graph of the associated functions

1.

$$F: \mathbb{R} \longrightarrow \mathbb{R} \\ x \mapsto 2^x + 1$$

$$[EF(F) = \mathbb{R}]$$

GNU01: $y = 2^x + 1$

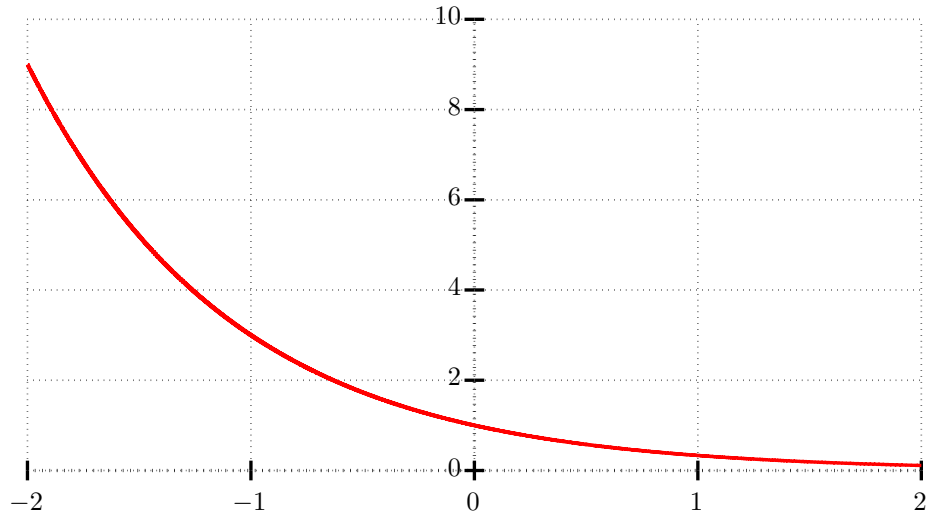


2.

$$F: \mathbb{R} \longrightarrow \mathbb{R}$$
$$x \mapsto \left(\frac{1}{3}\right)^x$$

$[EF(F) = \mathbb{R}]$

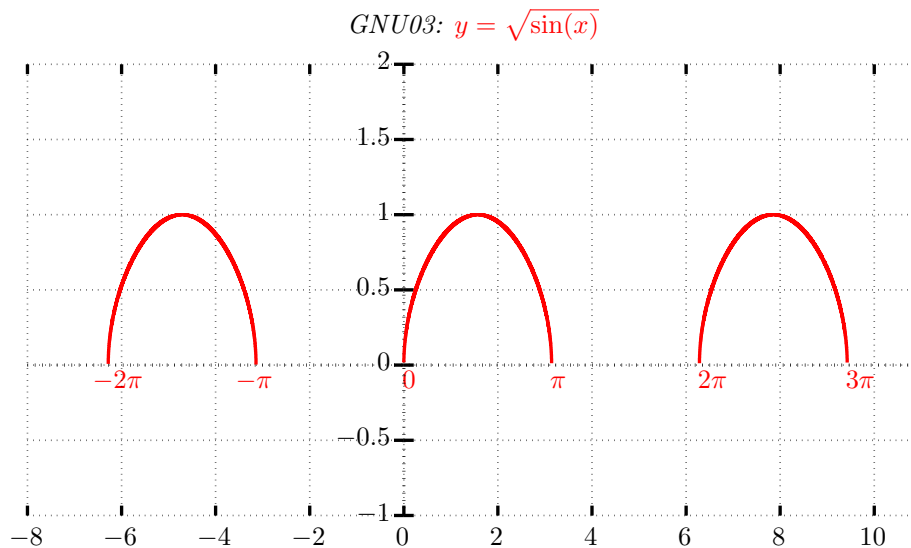
GNU02: $y = \left(\frac{1}{3}\right)^x$



3.

$$F: \mathbb{R} \longrightarrow \mathbb{R}$$
$$x \mapsto \sqrt{\sin(x)}$$

$$[EF(F) = \bigcup_{k \in \mathbb{Z}} [2k\pi, \pi + 2k\pi]]$$

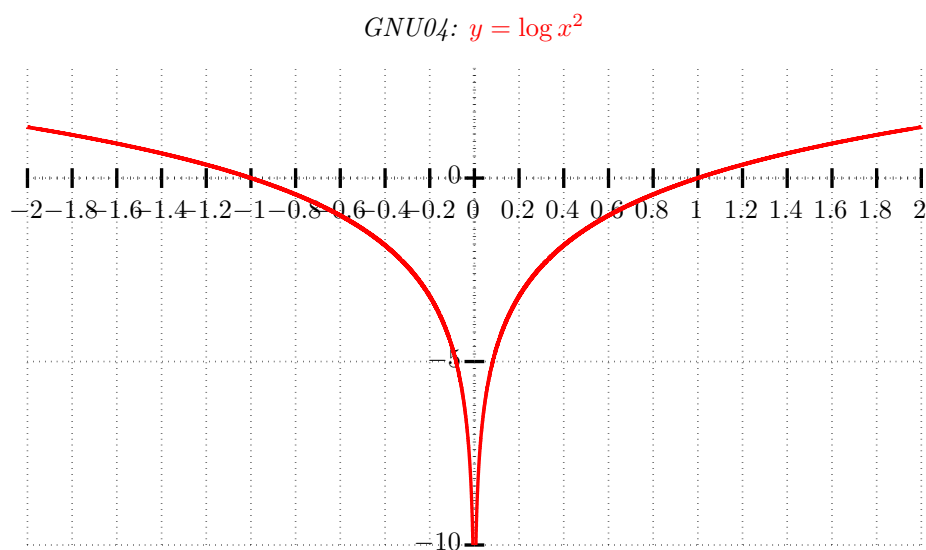


4.

$$F: \mathbb{R} \longrightarrow \mathbb{R}$$

$$x \longmapsto \log x^2$$

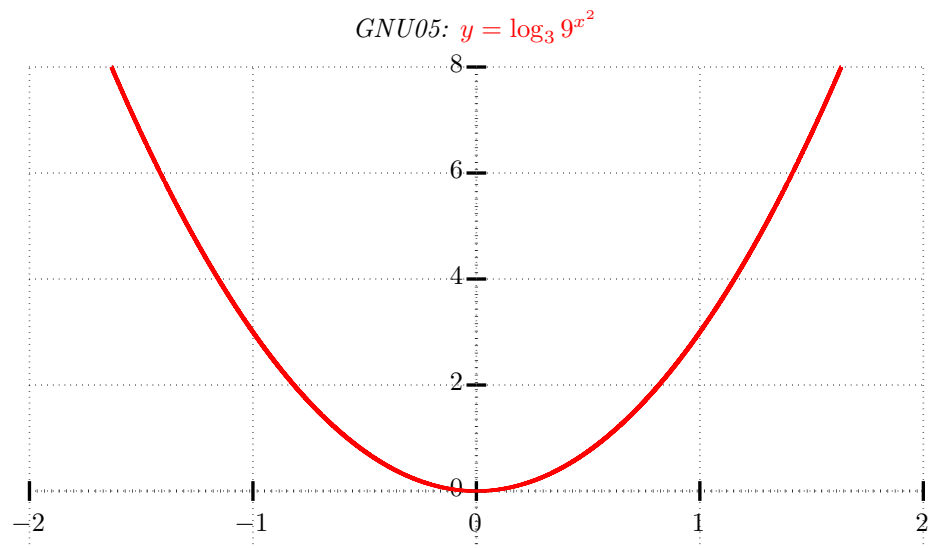
$$[EF(F) = \mathbb{R} - \{0\}]$$



5.

$$\begin{aligned} F : \mathbb{R} &\longrightarrow \mathbb{R} \\ x &\longmapsto \log_3 9^{(x^2)} \end{aligned}$$

$$[EF(F) = \mathbb{R}]$$

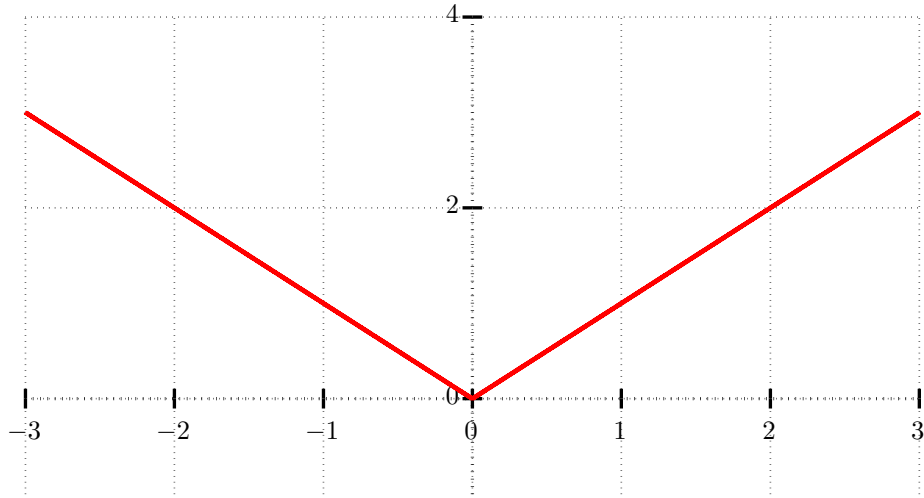


6.

$$\begin{aligned} F : \mathbb{R} &\longrightarrow \mathbb{R} \\ x &\longmapsto |x| \end{aligned}$$

$$[EF(F) = \mathbb{R}]$$

GNU06: $y = |x|$



7.

$$\begin{aligned} F: \mathbb{R} &\longrightarrow \mathbb{R} \\ x &\longmapsto |x^2| \end{aligned}$$

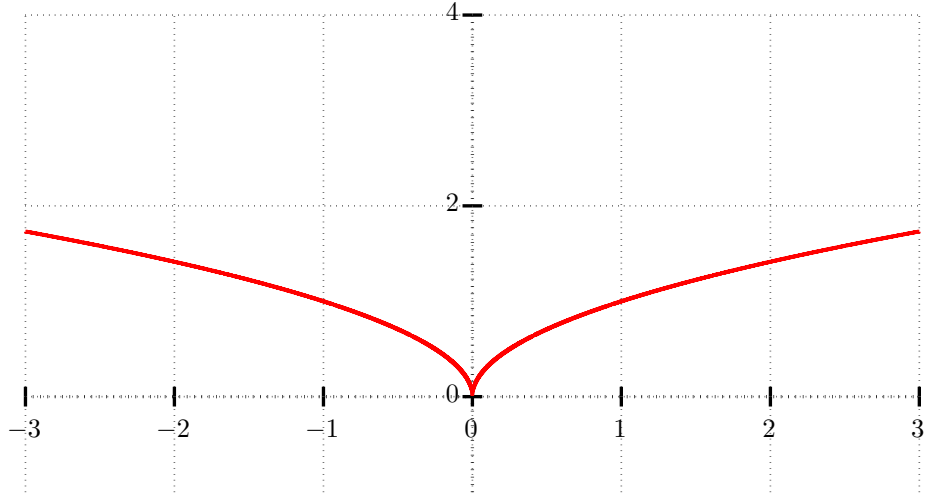
[The same as $y = x^2$]

8.

$$\begin{aligned} F: \mathbb{R} &\longrightarrow \mathbb{R} \\ x &\longmapsto \sqrt{|x|} \end{aligned}$$

[$EF(F) = \mathbb{R}$]

GNU07: $y = \sqrt{|x|}$

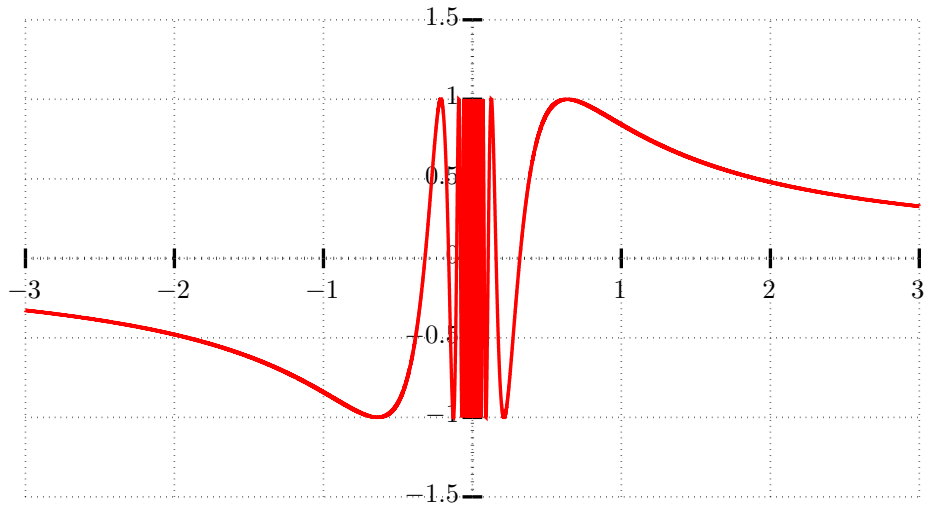


9.

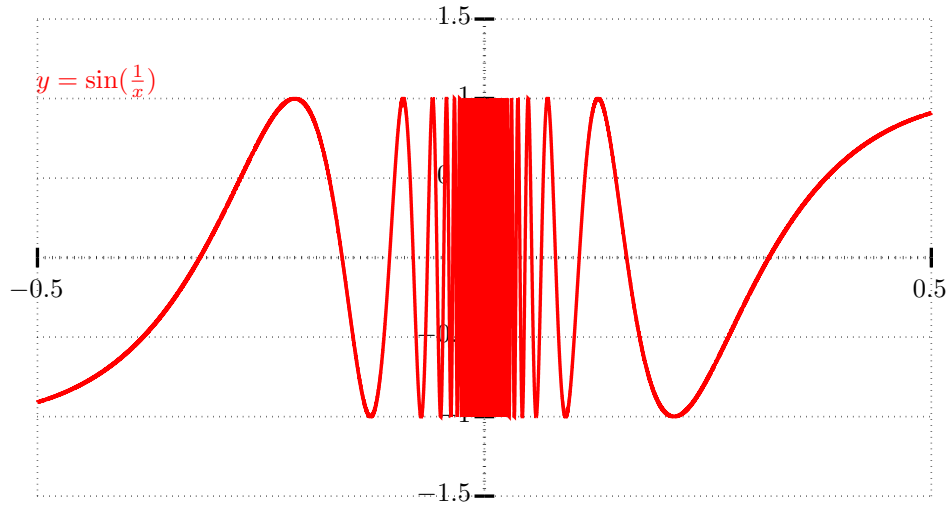
$$F: \mathbb{R} \rightarrow \mathbb{R}$$
$$x \mapsto \sin\left(\frac{1}{x}\right)$$

$$[EF(F) = \mathbb{R} - \{0\}]$$

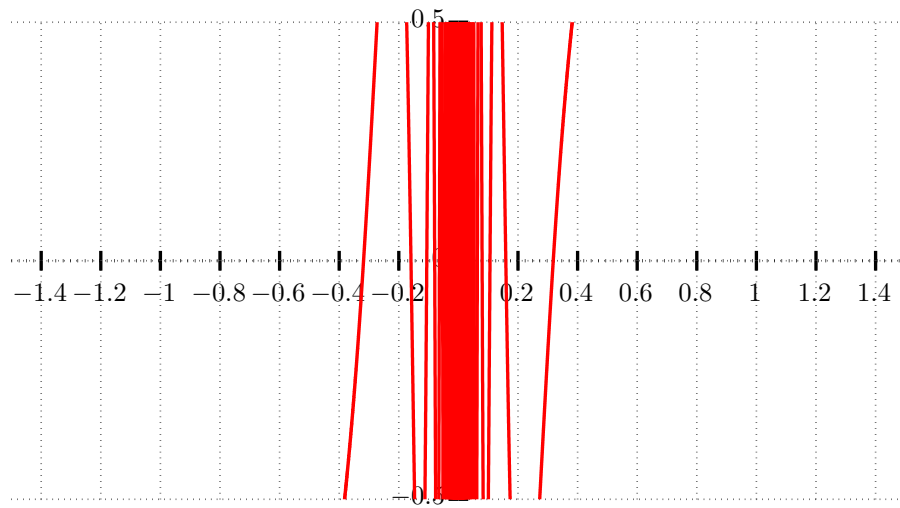
GNU08: $y = \sin\left(\frac{1}{x}\right)$



GNU09: $y = \sin(\frac{1}{x})$:more details



GNU09b: $y = \sin(\frac{1}{x})$:even more details

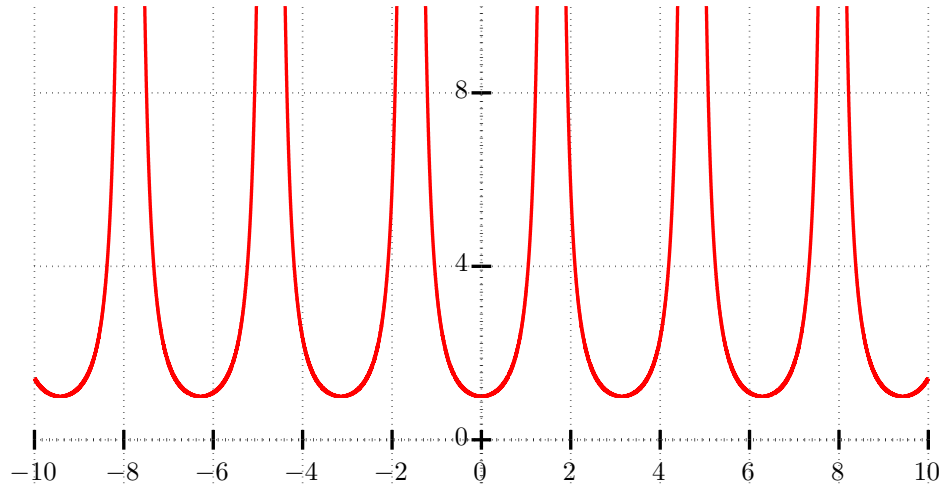


10.

$$F: \mathbb{R} \longrightarrow \mathbb{R}$$
$$x \mapsto \tan^2(x) + 1$$

$[EF(F) = \mathbb{R} - \{k\pi/2 \mid k \in \mathbb{Z}\}$ - the same as $y = \tan(x)$]

GNU10: $y = \tan^2(x) + 1$



Exercise 3. Determine, if possible, two periods and THE PERIOD of the functions

1.

$$\begin{aligned} F : \mathbb{R} &\longrightarrow \mathbb{R} \\ x &\mapsto \sin(x) + \cos(x) \end{aligned}$$

[Two periods: $2\pi, 4\pi$. PERIOD 2π]

2.

$$\begin{aligned} F : \mathbb{R} &\longrightarrow \mathbb{R} \\ x &\mapsto \sin(x) \cos(x) \end{aligned}$$

[Two periods: $2\pi, 4\pi$. PERIOD π]

3.

$$\begin{aligned} F : \mathbb{R} &\longrightarrow \mathbb{R} \\ x &\mapsto \tan(2x) \end{aligned}$$

[Two periods: $2\pi, 4\pi$. PERIOD $\pi/2$]

4.

$$\begin{aligned} F : \mathbb{R} &\longrightarrow \mathbb{R} \\ x &\mapsto x + \sin(x) \end{aligned}$$

[There are no periods]

5.

$$\begin{aligned} F : \mathbb{R} &\longrightarrow \mathbb{R} \\ x &\mapsto 1 \end{aligned}$$

[Two periods: 2, 3. There is no PERIOD]

6.

$$\begin{aligned} F: \mathbb{R} &\longrightarrow \mathbb{R} \\ x &\mapsto \sin(x) + \tan(x) \end{aligned}$$

[Two periods: $2\pi, 4\pi$. PERIOD 2π]

7.

$$\begin{aligned} F: \mathbb{R} &\longrightarrow \mathbb{R} \\ x &\mapsto \sin^2(x) + \cos^2(x) \end{aligned}$$

[Two periods: 2, 3. There is no PERIOD]