FCS
Math: Functions
Exercises

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Exercise 1. *Solve the following equations*

1. $\sqrt{2x-1} = \frac{1}{2}x$
2. $\sqrt{x^2+1} = x+1$
3. $\sqrt{x^4-x} = x^2$
4. $2^{x+1} + 4^x = 8$
5. $3^{\sqrt{2^x}} = 1$

Exercise 2. *Are the following functions invertible? If not, restrict the domain/codomain to build an invertible function. Give the explicit formula for the inverse.*

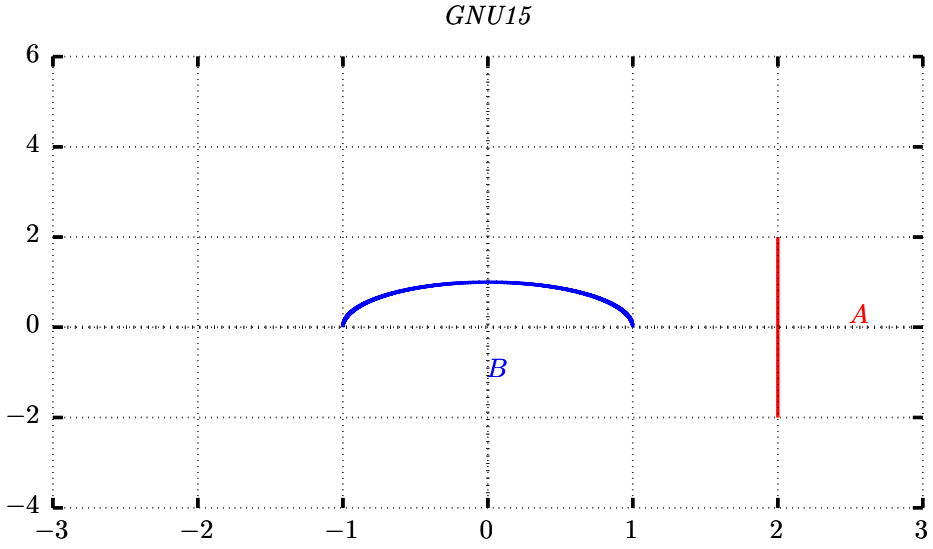
1. $\begin{array}{rccc} F: & \mathbb{R} & \longrightarrow & \mathbb{R} \\ & x & \mapsto & 3^{x+2} \end{array}$.
2. $\begin{array}{rccc} F: & \mathbb{R} & \longrightarrow & \mathbb{R} \\ & x & \mapsto & 3^{2x-1} \end{array}$.
3. $\begin{array}{rccc} F: & \mathbb{R} & \longrightarrow & \mathbb{R} \\ & x & \mapsto & \log_5(\mid x \mid) \end{array}$
4. $\begin{array}{rccc} F: & \mathbb{R} & \longrightarrow & \mathbb{R} \\ & x & \mapsto & \arctan(x) \end{array}$
5. $\begin{array}{rccc} F: & \mathbb{R} & \longrightarrow & \mathbb{R} \\ & x & \mapsto & \arctan(x^2) \end{array}$
6. $\begin{array}{rccc} F: & \mathbb{R} & \longrightarrow & \mathbb{R} \\ & x & \mapsto & \arctan(x-2)+1 \end{array}$
7. $\begin{array}{rccc} F: & \mathbb{R} & \longrightarrow & \mathbb{R} \\ & x & \mapsto & \arctan(3^x) \end{array}$

8. $$\begin{array}{rccc} F: & \mathbb{R} & \longrightarrow & \mathbb{R} \\ & x & \mapsto & \arctan(\sqrt{x}) \end{array}$$

Exercise 3. *Is there an invertible function between the following sets? If the answer is affermative, give the explicit function, by formula or an explict description.*

1. $A = \{5n - 2 \mid n \in \mathbb{N}\}$, $B = \{k - 4 \mid k \in \mathbb{N}\}$.
2. $A = \{n^2 \mid n \in \mathbb{N}\}$, $B = \{k^3 \mid k \in \mathbb{N}\}$.
3. $A = \{n^2 \mid n \in \mathbb{N}\}$, $B = \{k^3 \mid k \in \mathbb{Z}\}$.
4. $A = \{n^2 \mid n \in \mathbb{Z}\}$, $B = \{k^3 \mid k \in \mathbb{Z}\}$.
5. $A = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 = 1\}$, $B = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 = 4\}$.

Exercise 4. *Do the subsets A, B in $\mathbb{R}^2$ have the same cardinality?*



Exercise 5. *Do the subsets* $A = (0,1) \times \{1\}$, $B = \mathbb{R} \times \{2\}$ *in* $\mathbb{R}^2$ *have the same cardinality?*

Remark that $A = \{(t,1) \in \mathbb{R}^2 \mid t \in (0,1)\}$ *and* $B = \{(t,2) \in \mathbb{R}^2 \mid t \in \mathbb{R}\}$. *Graphically,*