$F: A \to A$: id_A OR IDENTITY FUNCTION OF A

$a \rightsquigarrow a$

$f: \mathbb{R} \to \mathbb{R}$

$x \rightsquigarrow x$

$g: \mathbb{R} \to \mathbb{R}$ 0-FUNCTION $\quad c: \mathbb{R} \to \mathbb{R}$

$x \rightsquigarrow 0 \qquad x \rightsquigarrow c$

$2^x = 3 \longmapsto$ X= CONSTANT $\qquad h(x) = g(x)$

$h, g: \mathbb{R} \to \mathbb{R}$

$3x - 1 = 4 \to 3x = 5 \qquad x = \frac{5}{3}$

$\log_{12}(2^x) = \log_{12}(3)$

?

$3 > 0$

$2^x > 0$

ARE THE SAME?

ARE EQUIVALENT?

$2^x = 3$ $\Leftrightarrow$ HAVE THE SAME ANSWER?

$\log_{12} 2^x = \log_{12} 3 \iff x \cdot \log_{12} 2 = \log_2 3$ (1)

$x = \log_2 3$

$f(x) = g(x) \iff h \circ f(x) = h \circ g(x)$

I: THIS HAS TO BE POSSIBLE

II: THIS HAS TO BE USEFUL

$f(x) = x^2 \quad g(x) = -3$

$h(x) = \sqrt{x}$

$x^2 = -3 \not\Rightarrow \sqrt{x^2} = \sqrt{-3}$

$\sqrt{}: \mathbb{R}_0^+ \to \mathbb{R}$

$x \rightsquigarrow \sqrt{x}$

$X = 3$ sol : 3 $f(x) = x$ $g(x) = 3$ $h(x) = x^2$

$X^2 = 9$ EQ?

sol : ± 3 → sol $\{x \in \mathbb{R} \mid x = 3\}$

NOT EQ → sol $\{x \in \mathbb{R} \mid x = \pm 3\}$

CHECK : 3 is sol $x^2 = 9$

$3 = 3$ OK 3 is sol. $X = ?$

-3 is sol of $x^2 = 9$

$-3 \neq 3$ NO -3 NOT sol $X = 3$

I st QUESTION : WHEN WE ADD SOLUTIONS?

II nd QUESTION : WHEN WE 'SOLVE' THE EQUATION 'SIMPLIFY'

$2^x = 3 \longrightarrow \log_2 2^x = \log_2 3 \longrightarrow x = \log_2 3$

x

$3x - 2 = 0 \longrightarrow x = \frac{2}{3}$??

$\sqrt{x} = 2$

I : $\exists \sqrt{x}$ $EF(\sqrt{x}) = \boxed{x \geq 0}$

II : APPLY FUNC. $f(x) = x^2$ $f : \mathbb{R} \to \mathbb{R}$, $x \mapsto x^2$

$(\sqrt{x})^2 = 2^2 \to x = 4$

$h(x) = \sqrt{x}$

$x^2 = 2 \to \sqrt{x^2} = \sqrt{2}$ $h : \mathbb{R}_0^+ \to \mathbb{R}$, $x \mapsto \sqrt{x}$

≥ 0 ≥ 0

x $x = \sqrt{2}$

sol $x = \pm\sqrt{2}$ sol $x = \sqrt{2}$

DEF $F: A \to A$ IF THERE IS $G: A \to A$ S.t.

$$F \circ G = G \circ F = Id_A$$

WE SAY THAT G IS THE INVERSE OF F

$$G = F^{-1} \quad [\, G = F^{-1} \Rightarrow F = G^{-1} \,]$$

$\log_2(2^x) = x$

$\log_2 x$ IS THE INVERSE OF 2^x

$f: \mathbb{R} \to \mathbb{R}$

$x \rightsquigarrow 2x - 3$

$g = f^{-1}: \mathbb{R} \to \mathbb{R}$

$x \rightsquigarrow ??$

$\forall x \; f \circ g(x) = x = g \circ f(x)$

$2x - 3 = y \xrightarrow{\text{SOLVE FOR } x} x = \frac{y+3}{2}$

$f^{-1}: \mathbb{R} \to \mathbb{R}$

$x \rightsquigarrow \frac{x+3}{2}$

$\forall x$ WE HAVE TO CHECK

$f^{-1} \circ f(x) = x$ AND $f \circ f^{-1}(x) = x$

$f \circ f^{-1}(x) = f\left(\frac{x+3}{2}\right) = 2 \cdot \frac{x+3}{2} - 3 = x$ OK

$f^{-1} \circ f(x) = f^{-1}(2x-3) = \frac{2x-3+3}{2} = x$ OK

$f(x) = g(x)$

$f^{-1} \circ f(x) = f^{-1} \circ g(x)$

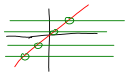
$x = f^{-1} \circ g(x)$

I* : WHEN $f: A \to A$ IS INVERTIBLE ?

PROP : $f: A \to A$ IS INVERTIBLE

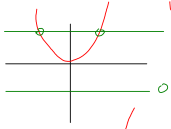
$\Leftrightarrow \forall b \in A \ \exists! \ a \in A$ s.t. $f(a) = b$

$\Leftrightarrow \forall b \in A$ THE GRAPH OF f IS INTERSECTED ONCE by $y = b$



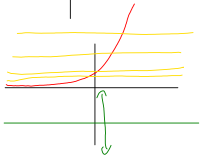
LINES ARE INVERTIBLE

PARABOLAS ARE INV ? NOT



2 WT

0 WT



$f: \mathbb{R} \to \mathbb{R}$

$x \rightsquigarrow 2^x$

NOT INV

IS INJECTIVE

$f: \mathbb{R} \to \mathbb{R}^+$

$x \rightsquigarrow 2^x$

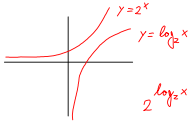
IS INV

0 WT !!

DEF $F: A \to B$ IS INVERTIBLE

IF THERE IS $G: B \to A$ s.t. $G \circ F = id_A$

$F \circ G = id_B$



$2^{\log_2 x} = \log_2 2^x = x$

$x > 0$?? Always $\forall x$

$2^x : \mathbb{R} \to \mathbb{R}^+$

$x \rightsquigarrow 2^x$

$2^{\log_2 x} = x$ $x > 0$

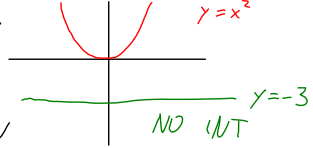
$\log_2 x : \mathbb{R}^+ \to \mathbb{R}$

$x \rightsquigarrow \log_2 x$

$\log_2 2^x = x$ $\forall x$

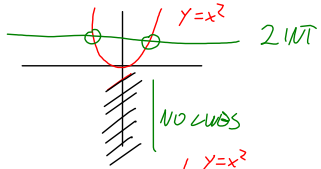
$f: \mathbb{R} \to \mathbb{R}$ NOT INV

$x \rightsquigarrow x^2$



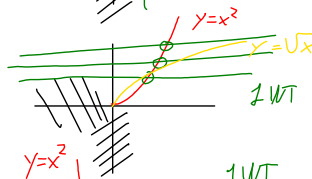
$f: \mathbb{R} \to \mathbb{R}_0^+$ NOT INV

$x \rightsquigarrow x^2$



$f: \mathbb{R}_0^+ \to \mathbb{R}_0^+$ INV!!

$x \rightsquigarrow x^2$

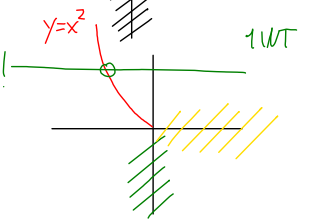


$g: \mathbb{R}_0^+ \to \mathbb{R}_0^+$

$x \rightsquigarrow \sqrt{x}$

$f: \mathbb{R}_0^+ \to \mathbb{R}_0^+$ INV!!

$x \rightsquigarrow x^2$



$g: \mathbb{R}_0^+ \to \mathbb{R}_0^-$

$x \rightsquigarrow -\sqrt{x}$

x^2

$-3 \to 9 \to -3$

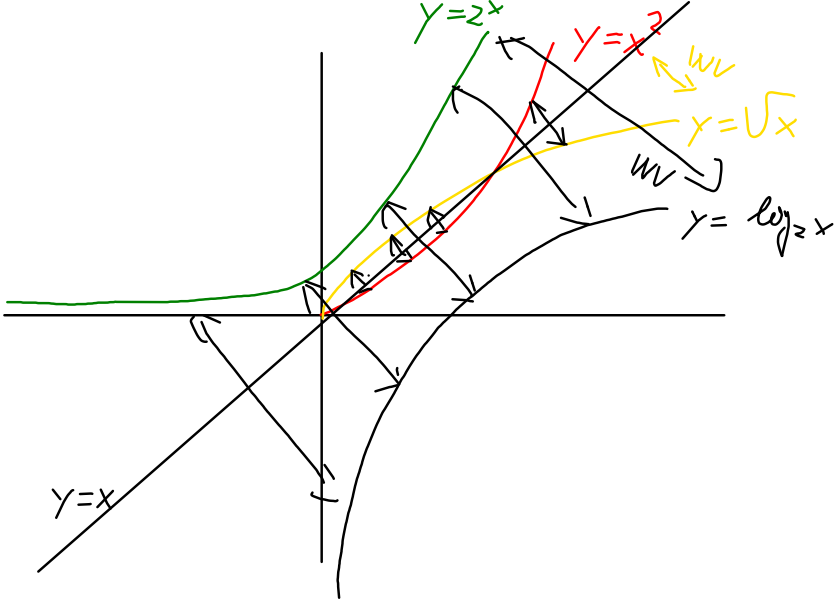
$-4 \to 16 \to -4$

$-2 \to 4 \to -2$

$\forall x \quad g \circ f(x) = x$

$-\sqrt{x^2} = x$

THIS IS TRUE $x \leq 0$



IF YOU HAVE THE GRAPH OF f AND f IS INVERTIBLE THE GRAPH OF f^{-1} IS THE SYMMETRIC OF GR(f) W.R.T. THE LINE $y=x$

$F: A \to B$ IFF $\overset{?}{\Longrightarrow}$ $F^{-1}: B \to A$

$a \rightsquigarrow F(a)$ $\qquad$ $y \rightsquigarrow F^{-1}(y)$

THE INVERSE OF F

$f: \mathbb{R} \to \mathbb{R}$ $\longrightarrow$ $f^{-1}: \mathbb{R} \to \mathbb{R}$

$x \rightsquigarrow 2x-3$ $\qquad$ $y \rightsquigarrow \frac{y+3}{2}$

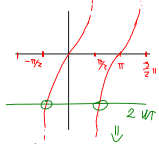
SOLVE FOR x

$2x-3=y$

$f : \mathbb{R} - \{\frac{\pi}{2}\} \longrightarrow \mathbb{R}$

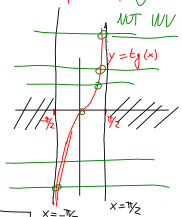
$x \rightsquigarrow tg(x)$

$f' = f|_{(-\pi/2, \pi/2)}$ TANGENT RESTRICTED TO



$f' : (-\pi/2, \pi/2) \longrightarrow \mathbb{R}$

$x \rightsquigarrow tg(x)$



$f'^{-1} : \mathbb{R} \longrightarrow (-\pi/2, \pi/2)$

ARCTAN (x)

$x \rightsquigarrow \text{ARCTAN}(x)$

$\text{TAN}(\frac{\pi}{4}) = \frac{\text{SIN}(\pi/4)}{\cos(\pi/4)} = \frac{\sqrt{2}/2}{\sqrt{2}/2} = 1$

$\text{ARCTAN}(1) = \pi/4$

$\text{TAN}(\pi/6) = \frac{\text{SIN}(\pi/6)}{\cos(\pi/6)} = \frac{1/2}{\sqrt{3}/2} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

$\text{ARCTAN}(\frac{\sqrt{3}}{3}) = \pi/6$

$\text{ARCTAN}(0) = 0$

$\text{TAN}(0) = 0 = \frac{\text{SIN}(0) = 0}{\cos(0) = 1} = 0$

LINES $\longleftrightarrow$ OK WE KNOW HOW TO INVERT

PARABOLAS $\longleftrightarrow$ " WHEN AND HOW

EXP $\rightsquigarrow$ LOG

LOG $\rightsquigarrow$ EXP

TAN $\rightsquigarrow$ ARCTAN

ABS NOT INV

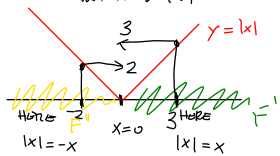
?

$|3x-2| = 5$

$f(x)$ $\quad$ $g(x)$

$h(x) : \mathbb{R} \to \mathbb{R}$

NOT OF $|x|$



$$|x| = \begin{cases} x & \text{IS } x \geq 0 \\ -x & \text{IS } x < 0 \end{cases}$$

$|-3| = 3$

$|3| = 3$

$F : \mathbb{R} \to \mathbb{R}$ NOT INV!

$x \rightsquigarrow |x|$

$F' : \mathbb{R}_0^+ \to \mathbb{R}_0^+$ IS INV

$x \rightsquigarrow |x| = x$

$F'' : \mathbb{R}_0^- \to \mathbb{R}_0^+$ IS INV

$\rightsquigarrow |x| = -x$

$|3x-2| = 5$

$\Updownarrow$

IF $\begin{cases} 3x-2 \geq 0 \\ 3x-2 = 5 \end{cases}$ APPLY G

IF $\begin{cases} 3x-2 < 0 \\ 2-3x = -5 \end{cases}$ APPLY H^{-1}

$-(2-3x) = 5$

$F : \mathbb{R}_0^+ \to \mathbb{R}_0^+$

$x \rightsquigarrow |x| = x$

INV IS $G : \mathbb{R}_0^+ \to \mathbb{R}_0^+$

$x \rightsquigarrow x$

$H : \mathbb{R}^- \to \mathbb{R}^+$

$x \rightsquigarrow |x| = -x$

INV $H^{-1} : \mathbb{R}_0^+ \to \mathbb{R}^-$

$x \rightsquigarrow -x$

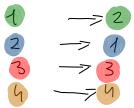
$F: A \to B$ WV $\iff \forall b \in B \; \exists! \, a \in A$ S.C. $F(a) = b$

$\iff \exists \; F^{-1}: B \to A$ S.T.

$F \circ F^{-1} = id \qquad F^{-1} \circ F = id$

IF F IS INVERTIBLE $\iff$ F IS A 1-TO-1 CORRESPONDENCE BETWEEN A AND B

$F: \{1,2,3,4\} \to \{1,2,3,4\}$ F IS WV



$A = \{a, b, c, d, \Delta, \square\}$
$1 \; 2 \; 3 \; 4 \; 5 \; 6$

A HAS 6 ELEMENTS IF THERE IS $F: \{1, \dots, 6\} \to A$ INVERTIBLE

d

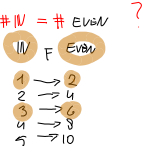
$\mathbb{N} = \{1,2,3,4,\dots\}$ $\#\mathbb{N}$

EVEN $= \{2,4,6,8,\dots\}$ $\#$ EVEN = NUMBER OF ELEMENTS OF EVEN

⇕ CARDINALITY

$\#\mathbb{N} = \#$ EVEN ?

IF $a \in \mathbb{N}$, $a \notin$ EVEN, $\mathbb{N} \supset$ EVEN $\Rightarrow \mathbb{N} \neq$ EVEN $\Rightarrow \#\mathbb{N} \neq \#$ EVEN



$F: \mathbb{N} \to$ EVEN $\quad n \rightsquigarrow 2n$

$F^{-1}:$ EVEN $\to \mathbb{N} \quad n \rightsquigarrow n/2$

JOE

1000 €

1 €

ADDS A COW IN TOP OF THE PILE OF COWS EVERY MONTH

JANE

1000 €

500 €

1000 PUTS THE COWS W THE PILE 1000 € EVERY MONTH ON TOP

← PUTS OUT 500 € EVERY MONTH FROM THE BOTTOM

SALARY MONEY EVERY MONTH

$\mathbb{N} \simeq$ EVEN $=$ ODD $\quad \mathbb{Z} \quad \mathbb{Q} \quad \mathbb{R} \quad \mathbb{C}$

SAME

CARDINALITY $\quad ? \quad ? \quad ? \quad ?$

$\mathbb{R}^2$