FCS
Math: Functions
Exercises

Massimo Caboara

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Exercise 1. *Solve the following equations, if possibile. If not, determine as much information you can about the solutions. In every case, determine the x for which the equation makes sense.*

1. $3^x = 3$
2. $2^x = 2^2$
3. $5^x = 5^{2x}$
4. $5^{x^2} = 5^{2x}$
5. $\log_3(x-1) = 3$
6. $\log_3(x-1) = x$
7. $3^x = x^2 - 4$
8. $3^x = x^2 - 4$
9. $\mid x \mid = 3$
10. $\mid x \mid = x$
11. $\mid x \mid = x^2$
12. $\sin x = \cos(x)$, *if* $x \in [0, 2\pi)$
13. $\sin x = \cos(x)$, *if* $x \in \mathbb{R}$
14. $\tan(x^2) = \tan(x)$, *if* $x \in (-\pi/2, \pi/2)$
15. $\sqrt{x^2} = \sqrt{x}$
16. $\tan x = 1$, *if* $x \in [0, 2\pi)$
17. $\sin x = 5/7$, *if* $x \in [0, 2\pi)$

18. $\tan x = 3$, *if* $x \in (-\pi/2, \pi/2)$

Exercise 2. *Are to following functions invertible? If the answer is yes, find the inverse if possibile*

1. *if* ODD *is the set of the odd positive numbers* $\begin{array}{rccc} F: & \mathbb{N} & \longrightarrow & \text{ODD} \\ & n & \mapsto & 2n+1 \end{array}$

2. $\begin{array}{rccc} F: & \mathbb{R} & \longrightarrow & \mathbb{R} \\ & x & \mapsto & -3x+4 \end{array}$

3. $\begin{array}{rccc} F: & [-1,+\infty) & \longrightarrow & \mathbb{R} \\ & x & \mapsto & \sqrt{x+1} \end{array}$

4. $\begin{array}{rccc} F: & \mathbb{R} & \longrightarrow & \mathbb{R} \\ & x & \mapsto & \mid 2x-1 \mid \end{array}$

Exercise 3. *Are to following functions invertible? If the answer is no, determine a restriction of the domain and/or codomain that produces and invertible function with the same formula*

1. $\begin{array}{rccc} F: & \mathbb{R} & \longrightarrow & \mathbb{R} \\ & x & \mapsto & x^2-4 \end{array}$

2. $\begin{array}{rccc} F: & \mathbb{R} & \longrightarrow & \mathbb{R} \\ & x & \mapsto & x^2+x \end{array}$

3. $\begin{array}{rccc} F: & [-1,+\infty) & \longrightarrow & \mathbb{R} \\ & x & \mapsto & \sqrt{x+1} \end{array}$

4. $\begin{array}{rccc} F: & \mathbb{R} & \longrightarrow & \mathbb{R} \\ & x & \mapsto & \sin(x) \end{array}$ *[Difficult]*

5. $\begin{array}{rccc} F: & \mathbb{R} & \longrightarrow & \mathbb{R} \\ & x & \mapsto & \cos(x) \end{array}$ *[Difficult]*

Exercise 4. *Is it possibile to find a one-to-one correspondence between the sets* $A = \{n \in \mathbb{N} \mid n \text{ is multiple of } 3\}$ *and* $B = \{n \in \mathbb{N} \mid n \text{ is multiple of } 4\}$

Exercise 5. *Is it possibile to find a one-to-one correspondence between the sets* $A = \{2n+2 \mid n \in \mathbb{N}\}$ *and* $B = \{n^2 \mid n \in \mathbb{N}\}$

Exercise 6. *Is the function* $F : \mathbb{R} \longrightarrow \mathbb{R}$ *whose graph in shown below invertible? If not, find some restrictions of*

the domanin/codomain that produces and invertible function

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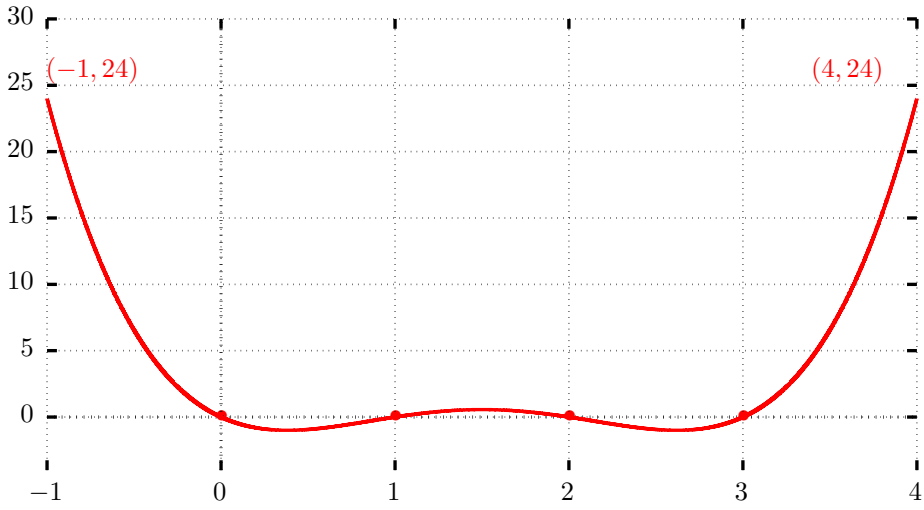


Exercise 7. *The sets* $\mathbb{N}, \mathbb{Z}$ *have the same cardinality?*

Exercise 8. *The sets* $\mathbb{N}, \mathbb{Q}$ *have the same cardinality? [Very difficult]*

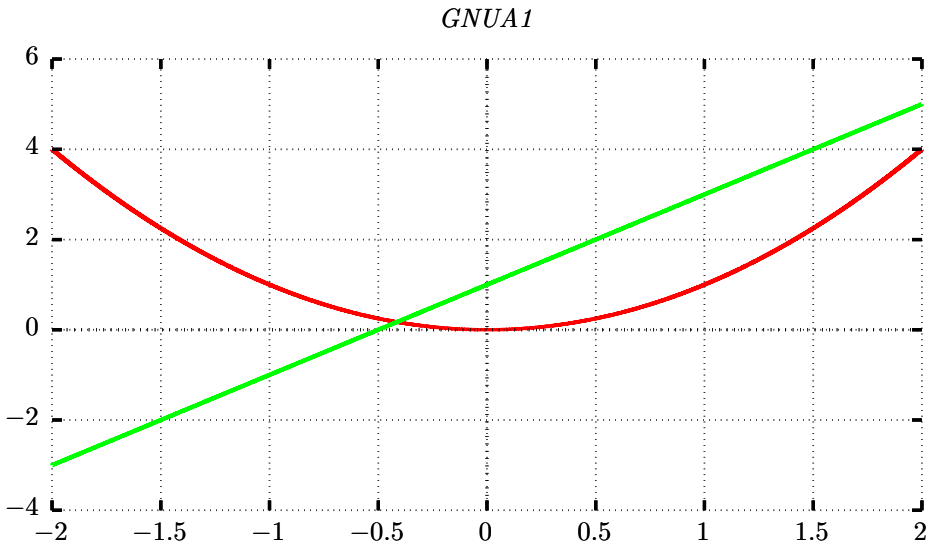
Exercise 9. *The sets* $\mathbb{N}, \mathbb{R}$ *have the same cardinality? [Very difficult]*

Exercise 10. *The sets* $\mathbb{N}, \mathbb{N}^2$ *have the same cardinality? [Very difficult]*

Exercise 11. *The sets* $\mathbb{N}, \mathbb{N}^3$ *have the same cardinality? [Very difficult]*

Exercise 12 (Hilbert hotel)**.** *We have an hotel with infinite rooms, all occupied. If a new customes comes, can we find a free room for him?*

Exercise 13. *Do the subsets* A, B *in* $\mathbb{R}^2$ *have the same cardinality?*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