$f : \mathbb{R} \longrightarrow \mathbb{R}$

$x \rightsquigarrow \sqrt{1/x}$

$\nexists f(0)$!

NOT A FUNCTION.

0 PB

-1 PB $\nexists f(-1)$

$$EF(f) = \begin{cases} x \neq 0 \\ 1/x \geq 0 \end{cases} = x > 0$$

$$EF(f) = \{ x \in \mathbb{R} \mid x > 0 \} = (0, +\infty)$$

$f : EF(f) \longrightarrow \mathbb{R}$

$x \rightsquigarrow \sqrt{1/x}$

$F : \mathbb{R} \to \mathbb{R}$ $\qquad EF(F) = \mathbb{R}$

$x \rightsquigarrow x^2 + 1$

$F : \mathbb{R} \longrightarrow \mathbb{R}$ $\qquad EF(F) = \emptyset$

$x \rightsquigarrow \sqrt{-1}$

Ex 6 $\quad F : \mathbb{R} \longrightarrow \mathbb{R}$ $\quad \nexists F(0)$

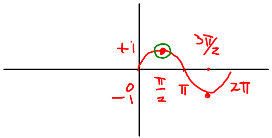
$x \rightsquigarrow \sqrt{\sin(x) - 1}$

$y = \sin(x)$

$-1 \leq \sin(x) \leq 1$

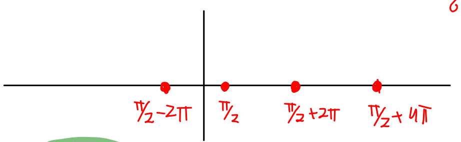
$-2 \leq \sin(x) - 1 \leq 0$

$\uparrow \quad x = \pi/2$



$\frac{\pi}{2} + k2\pi$

$EF(F) = \{\pi/2 + 2k\pi \mid k \in \mathbb{Z}\} = A$



GRAPH OF

$F' : A \longrightarrow \mathbb{R}$

$x \rightsquigarrow \sqrt{\sin(x) - 1}$

$G : A \longrightarrow \mathbb{R}$

$x \rightsquigarrow 0$

$F' \equiv G$ AS A FUNCTION

$\Updownarrow$

$\forall a \in A \quad F'(a) = G(a)$

$\quad\quad\quad\quad\quad\quad\quad \| \quad\quad\quad \|$

$\quad\quad\quad\quad\quad\quad\quad 0 \quad\quad\quad 0$

$A \quad [-\infty, 3] \quad (-1, 3]$

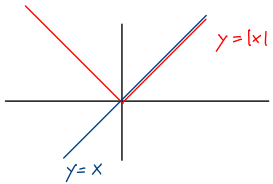
$F : \mathbb{R} \to \mathbb{R}$

$x \rightsquigarrow |x| = \begin{cases} x & \text{IF } x \geq 0 \\ -x & \text{IF } x < 0 \end{cases}$

$F(3) = 3$

$F(-3) = 3$

$F(0) = 0$



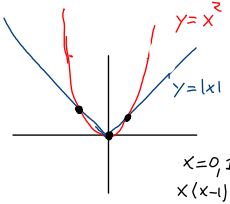
$|x| = x$?

$y = |x|$ $\quad$ $y = x$

$x \geq 0$

OR

$\{x \in \mathbb{R} \mid x \geq 0\}$ OR $[0, +\infty)$



$x^2 = |x|$?

$y = x^2$ $\quad$ $y = |x|$

$x = 0, 1$

$x(x-1) = 0$

SOL : $0, \pm 1$

$x^2 = |x| \iff \begin{cases} x^2 = x \\ x \geq 0 \end{cases} \cup \begin{cases} x^2 = -x \\ x < 0 \end{cases}$

$x^2 + x = 0$

$x(x+1) = 0$

$x = -1$

$|x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$

$\{0, 1\} \cup \{-1\}$

$\{0, \pm 1\}$

Ex $f: \mathbb{R} \to \mathbb{R}$, $x \rightsquigarrow 3$

$F: \mathbb{R} \to \mathbb{R}$, $x \rightsquigarrow \sin(x)$

2π, 4π, 10π ARE PERIODS

4 IS A PERIOD ? $\iff \forall x \in \mathbb{R}$ $F(x) = F(x+4)$

$3 = 3$

10 IS A PERIOD ✓

π IS A PERIOD ✓

$\mathbb{R}^+ = (0, +\infty)$

$p \in \mathbb{R}^+$ IS THE PERIOD OF $F \iff p = \min \{ q \in \mathbb{R}^+ \mid q \text{ IS A PERIOD} \}$

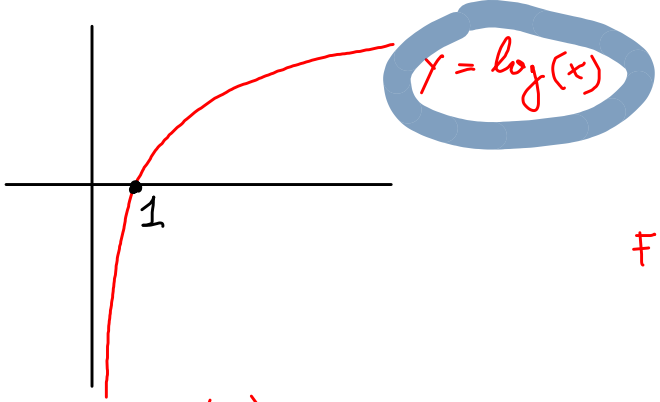
1 IS A PERIOD OF F

1 IS THE PERIOD OF F ? NO

0.5 IS A PERIOD

0.25

THERE IS NO $p \in \mathbb{R}^+$ THAT IS THE PERIOD OF F



$$F : \mathbb{R}^+ \longrightarrow \mathbb{R}$$
$$x \rightsquigarrow \log(x)$$

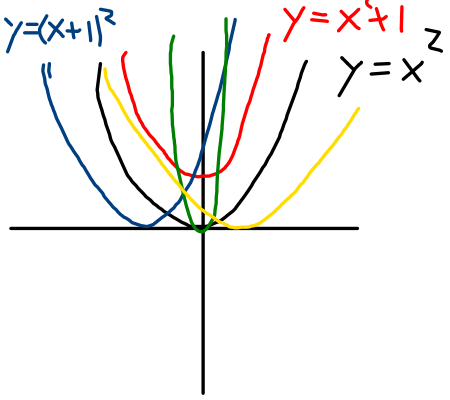
$$EF(F) = (0, +\infty) \qquad F(1) = 0$$

$$F : \mathbb{R} \longrightarrow \mathbb{R}$$
$$x \rightsquigarrow x^2,\ \sin(x),\ 3^x,\ \log(x)$$

$y = x^2$

$F: \mathbb{R} \to \mathbb{R}$

$x \rightsquigarrow F(x) = x^2$



$y = x^2 + 1$	$y = F(x) + 1$
$y = (x+1)^2$	$y = F(x+1)$
$y = 9x^2$	$y = F(3x)$
$y = (x-1)^2$	$y = F(x-1)$

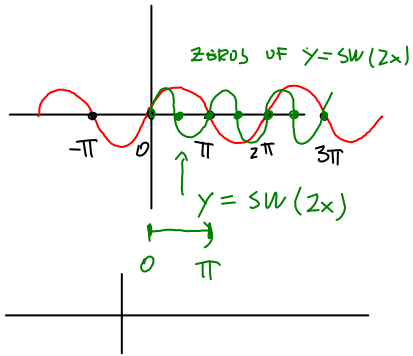
$$y = A\,F(Bx + C) + D$$

- A: AMPLIFIES ↕
- B: ↕
- C: ↔ LEFT / RIGHT
- D: ↕ UP / DOWN

$y = \sin(x)$

EXERCISE –

- $y = 3\sin(x)$
- $y = \sin(x+\pi)$
- $y = \sin(x) + 1$



$y = \sin(x)$

$y = A\,F(Bx + C) + D$

$y = \sin(2x)$

B

$y = \sin(Bx)$

HAS PERIOD $\frac{2\pi}{B}$

$y = \sin(2x)$

$\sin(2x) = 0$

$\sin(x) = 0$

$x = 0, \pi, 2\pi, \ldots$

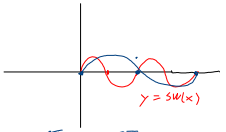
$+k\pi$

$2x = 0 \qquad x = 0$

$2x = \pi \qquad x = \pi/2$

$2x = 2\pi \qquad x = \pi$

ZEROS : $\pi/2 + \frac{k\pi}{2}$



$y = \sin(\frac{1}{2}x)$

PERIOD IS $\frac{2\pi}{\frac{1}{2}} = 4\pi$

$y = \sin(x) + \cos(x)$: 2π

$y = \sin(\frac{1}{2}x) + \cos(\frac{1}{4}x)$: 8π

4π 8π

$y = \sin(\frac{1}{2}x) + \cos(\frac{1}{3}x)$: ? 6π 12π ?

4π 6π

$y = F(x) + G(x)$? 6 12 ?

6 4 = LCM(4,6)

$H(x) = F(x) + G(x)$

$H(x+6) \overset{?}{=} H(x)$ $\forall x \in \mathbb{R}$? $\Leftrightarrow$ 6 IS A PERIOD

$F(x) + G(x)$

$F(x+6) + G(x+6)$

$F(x)$ $G(x)$

QUESTION: F, G PERIODIC OF PERIODS a, b

$F+G$ HAS PERIOD LCM(a,b) ?

$\sin^2(x)$: PERIOD 2π

$\cos^2(x)$ 2π

LCM(a,b) IS A PERIOD NOT NECESSARILY THE PERIOD

$\sin^2(x) + \cos^2(x) = 1$ NO PERIOD

$\sin(2x) + \cos(\sqrt{2}x)$ IS NOT PERIODIC

$$\sqrt{x} = 3x - 1$$

$$\Updownarrow ?$$

$$(\sqrt{x})^2 = (3x-1)^2$$

$$F : \mathbb{R} \to \mathbb{R}$$

$$x \rightsquigarrow x^2$$

$F : \mathbb{R} \to \mathbb{R}$ $\quad$ $G : \mathbb{R} \to \mathbb{R}$

$a \rightsquigarrow a^2 = F(a)$ $\quad$ $b \rightsquigarrow 2b-1 = G(b)$

F $\quad$ G

$: \mathbb{R} \longrightarrow \mathbb{R} \longrightarrow \mathbb{R}$

$x \rightsquigarrow x^2 \rightsquigarrow 2x^2-1$

$x \rightsquigarrow 2x-1$

$G(F(a)) = G(a^2) = 2a^2-1$

$H : \mathbb{R} \to \mathbb{R}$

$c \rightsquigarrow \frac{c}{2}+1$

$F(H(6))$

$F(\overset{\shortparallel}{\frac{6}{2}+1}) = \left(\frac{6}{2}+1\right)^2$

$F : \mathbb{R} \to \mathbb{R}$

$a \rightsquigarrow F(a)$

$G : \mathbb{R} \to \mathbb{R}$

$b \rightsquigarrow G(b)$

$G \circ F : \mathbb{R} \to \mathbb{R}$: COMPOSITION

$a \rightsquigarrow G(F(a))$

I) THERE IS A FUNCTION THAT PLAYS THE ROLE OF 0 W THE SUM ?

$F : \mathbb{R} \to \mathbb{R}$

$x \rightsquigarrow x^2$

$G : \mathbb{R} \to \mathbb{R}$

$x \rightsquigarrow x$

$\shortparallel$ IDENTITY FUNCTION

$F \circ G = F$

$G \circ F = F$ $\Big|$ TRUE $\forall F : \mathbb{R} \to \mathbb{R}$

III $3+5=5+3$

$F \circ G = G \circ F$? $\forall F, G : \mathbb{R} \to \mathbb{R}$

$F : \mathbb{R} \to \mathbb{R}$ $\quad x \rightsquigarrow x^2$

$G : \mathbb{R} \to \mathbb{R}$ $\quad x \rightsquigarrow 2x-1$

$F \circ G \equiv G \circ F$? $\overset{\text{DEF}}{\equiv} \iff \forall x \in \mathbb{R} \quad F \circ G(x) = G \circ F(x)$

$\Uparrow$ DEF $F \circ G$

$\forall x \in \mathbb{R} \quad F(G(x)) = G(F(x))$

$F(2x-1) \qquad G(x^2)$

$(2x-1)^2 \qquad 2x^2-1$

$4x^2-4x+1$ $\not\equiv$

III $F: A \to B$ $\quad$ $G: D \to C$

$a \rightsquigarrow F(a)$ $\quad$ $d \rightsquigarrow G(d)$

$G \circ F = G(F(a))$ ($F(a) \in B$)

WE KNOW WHAT THIS IS

$B \subseteq D$ IT IS NECESSARY ?

$F: \underset{A}{\mathbb{R}} \to \underset{B}{\mathbb{R}}$ $\quad$ $G: \underset{D}{\mathbb{R}_0^+} \to \underset{C}{\mathbb{R}}$

$x \rightsquigarrow |x|$ $\quad$ $x \rightsquigarrow \sqrt{x}$

$\mathbb{R} \not\subseteq \mathbb{R}_0^+$

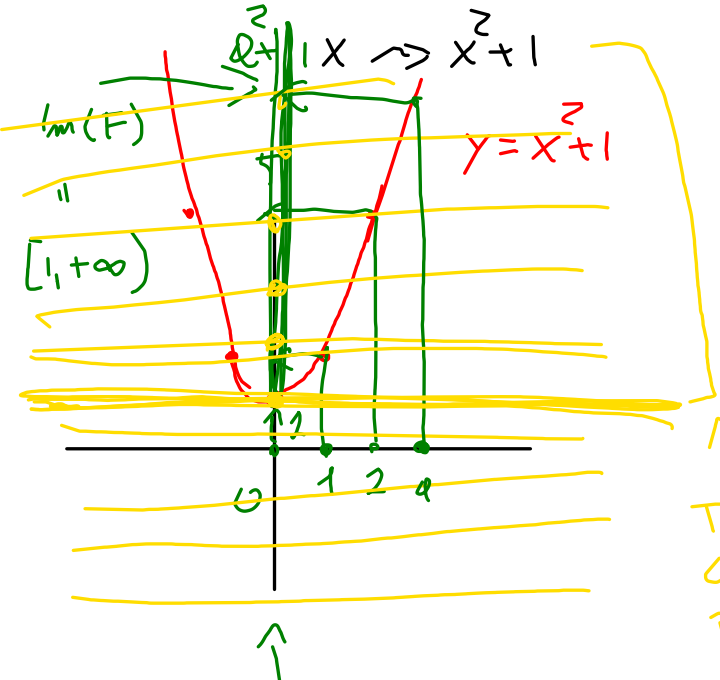
$\sqrt{|x|}$ IT IS MEANINGFUL.

DEF $F: A \to B$

$\operatorname{Im} F = \{ F(a) \mid a \in A \} \subseteq B$

$G \circ F$ IS MEANINGFUL IF $\operatorname{Im} F \subseteq D$

Ex $F : \mathbb{R} \to \mathbb{R}$



THOSE LINES GIVE YOU THE IMAGE

$\mathrm{Im}(F) \neq \mathbb{R} \ni -1$

$-1 \notin \mathrm{Im}(F)$

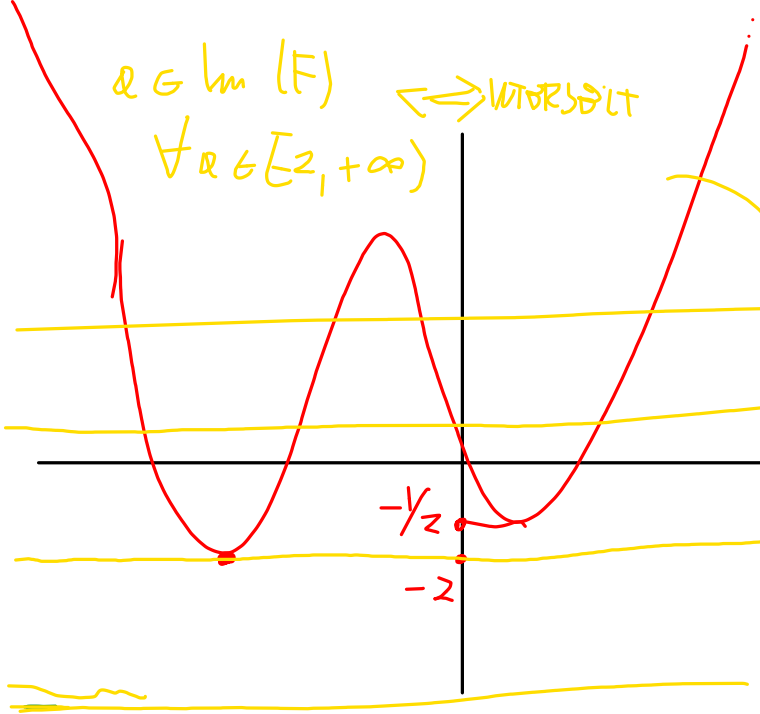
$=$

$\mathrm{Im}(F) = \{ x^2+1 \mid x \in \mathbb{R} \}$

$-1 \notin \mathbb{R} \iff x^2+1 = -1$

$x^2 = -2$

NO SOLUTION



$y = F(x)$

$F : \mathbb{R} \to \mathbb{R}$

$x \rightsquigarrow F(x)$

$a \geq -2$
$y = a$

$-2 \in \mathrm{Im}(F)$

$\Updownarrow$

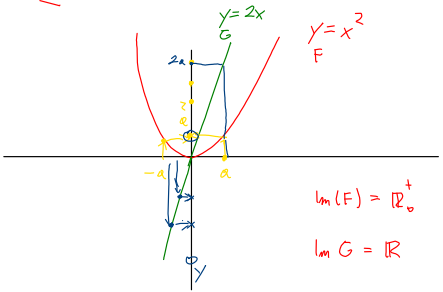
$y = -2$ INTERSECTS

$\mathrm{Im}(F) = [-2, +\infty)$

$y = -3$ NO INTER

$\Updownarrow$

$-3 \notin \mathrm{Im}(F)$



DEF : $F : A \to B$ FUNCTION

F IS WSECTIVE (ONE-TO-ONE)

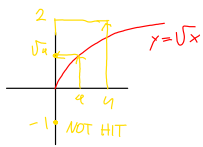
IF $\forall b \in B \quad \exists! a \in A$ s.t. $F(a) = b$

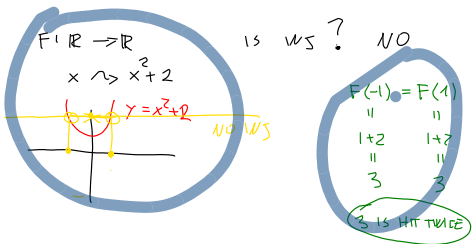
OR ~~$\exists$~~ $a \in A$ s.t. $F(a) = b$

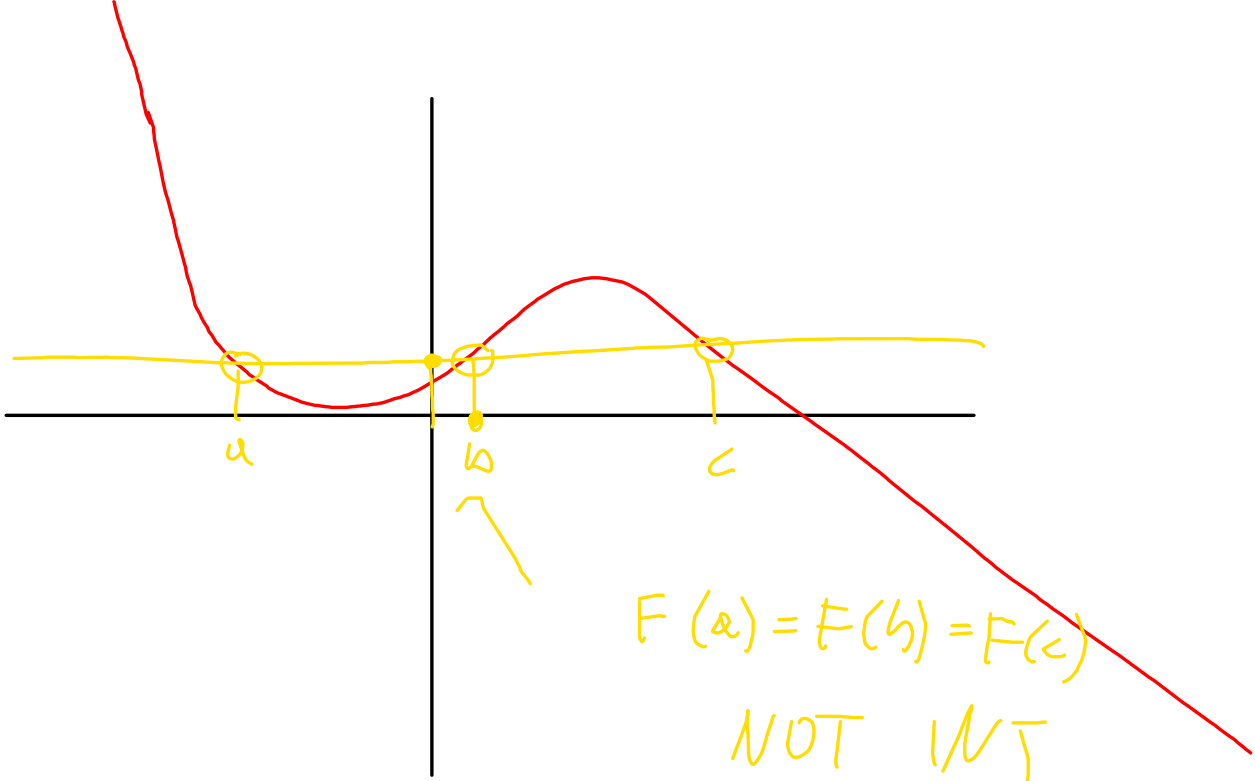
Ex $F : \mathbb{R}_0^+ \to \mathbb{R}$

$x \rightsquigarrow \sqrt{x}$

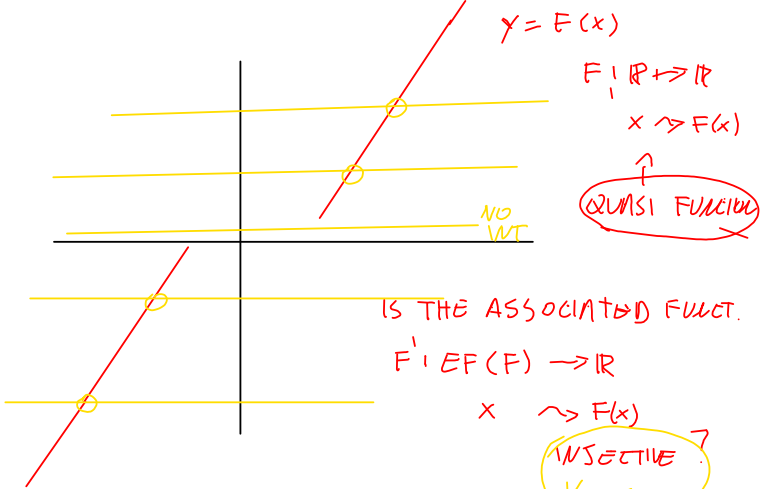
WSECTIVE






a
b
c
F(a) = F(b) = F(c)
NOT INJ


$y = \log(x)$
$F : \mathbb{R}^+ \to \mathbb{R}$
$x \rightsquigarrow \log(x)$



$Y = F(x)$

$F : \mathbb{R} \to \mathbb{R}$

$x \rightsquigarrow F(x)$

QUASI FUNCTION

IS THE ASSOCIATED FUNCT.

$F' : EF(F) \to \mathbb{R}$

$x \rightsquigarrow F(x)$

INJECTIVE ?

YES

$3x-1 = 2^x + 1$

$\Updownarrow$ THE EQUATIONS ARE EQ. IF F IS INJECTIVE

$F(3x-1) = F(2^x+1)$