

FCS
Math: Functions
Exercises

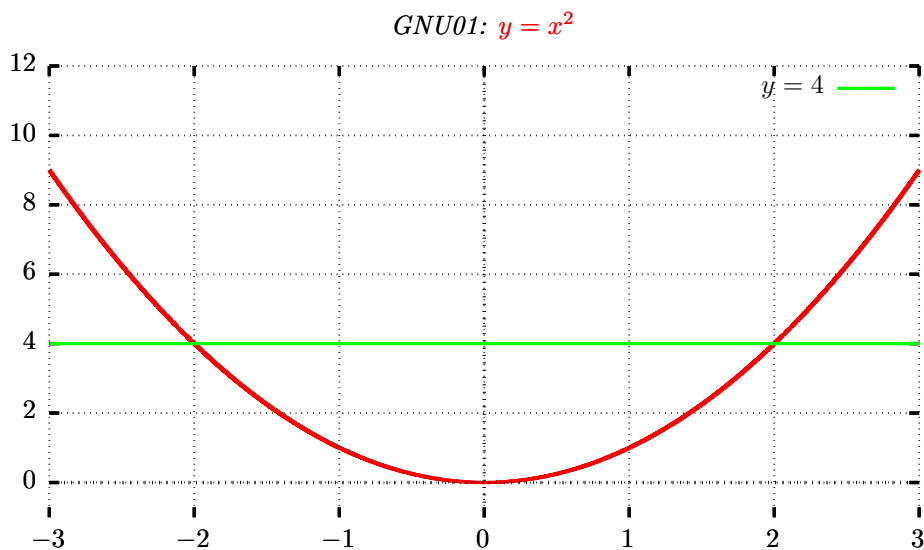
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Exercise 1. Determine if the following functions are injective, drawing the graphs

1.

$$F : \mathbb{R} \longrightarrow \mathbb{R} \\ x \mapsto F(x) = x^2$$

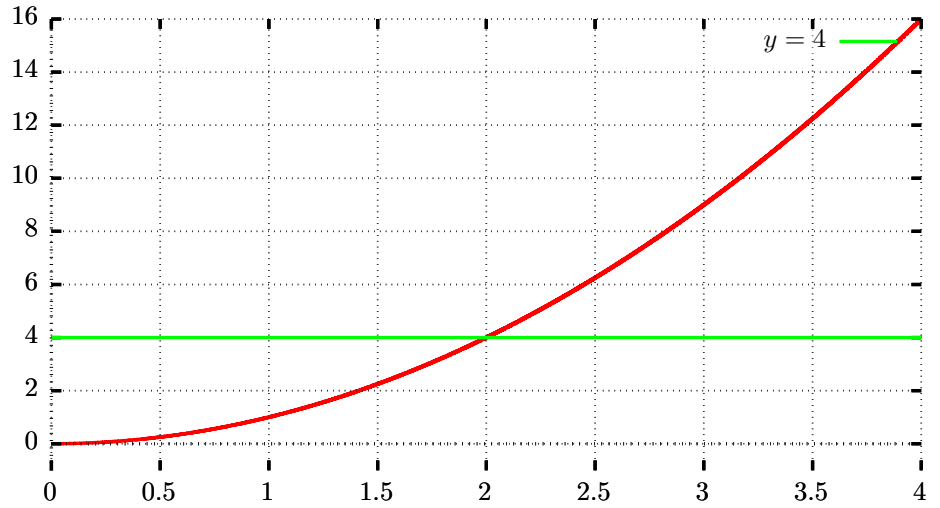


Non injective

2.

$$F : \mathbb{R}_0^+ \longrightarrow \mathbb{R} \\ x \mapsto F(x) = x^2$$

GNU02: $y = x^2$

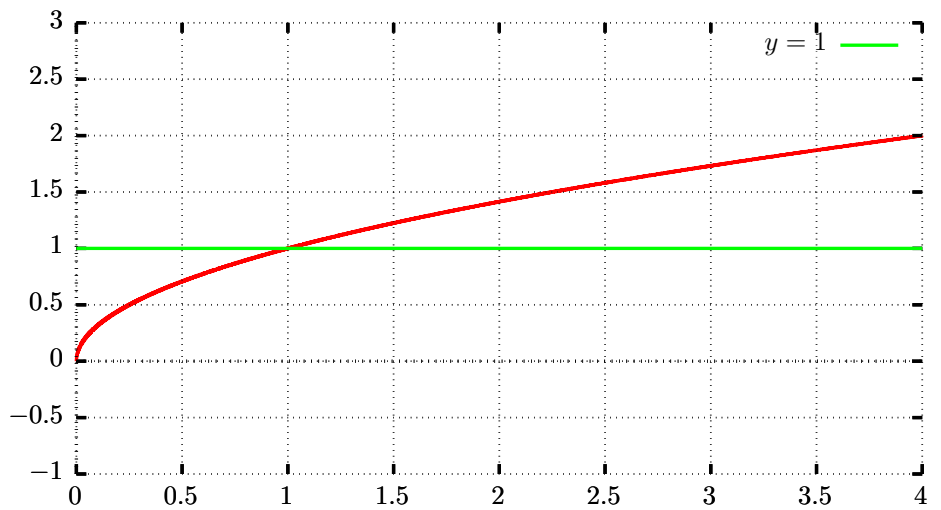


Injective

3.

$$F : \mathbb{R}_0^+ \longrightarrow \mathbb{R} \\ x \mapsto F(x) = \sqrt{x}$$

GNU03: $y = \sqrt{x}$

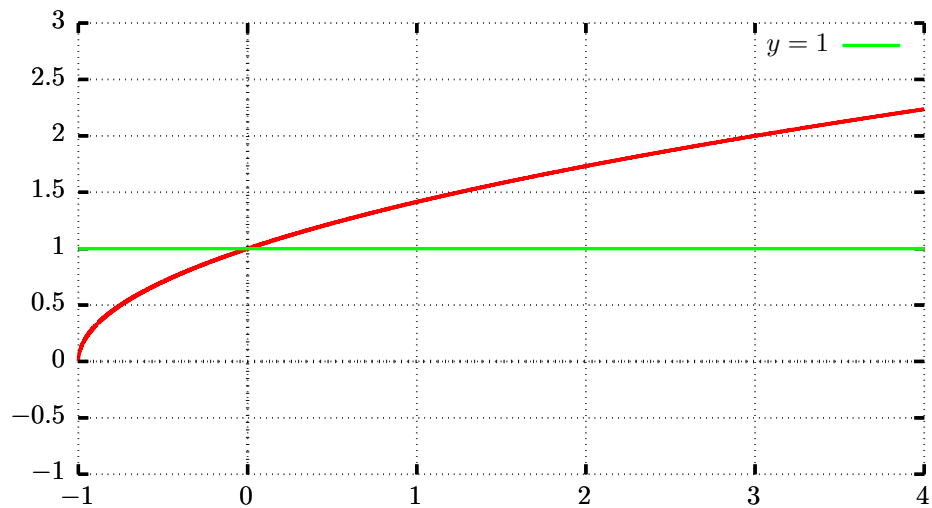


Injective

4.

$$F : [-1, +\infty) \longrightarrow \mathbb{R} \\ x \mapsto F(x) = \sqrt{x+1}$$

$$\text{GNU04: } y = \sqrt{x+1}$$



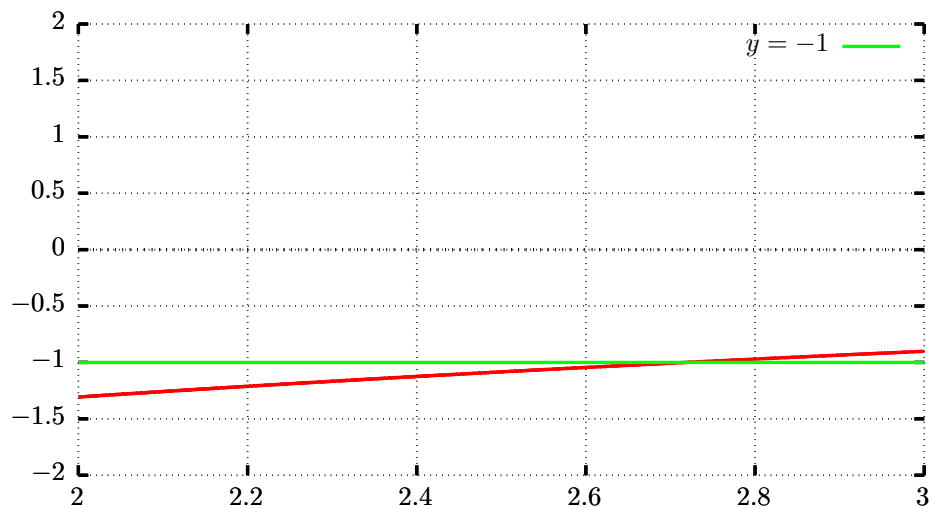
Injective

5.

$$F : [2, +\infty] \longrightarrow \mathbb{R}$$

$$x \mapsto F(x) = \log(x) - 2$$

$$\text{GNU05: } y = \log(x) - 2$$



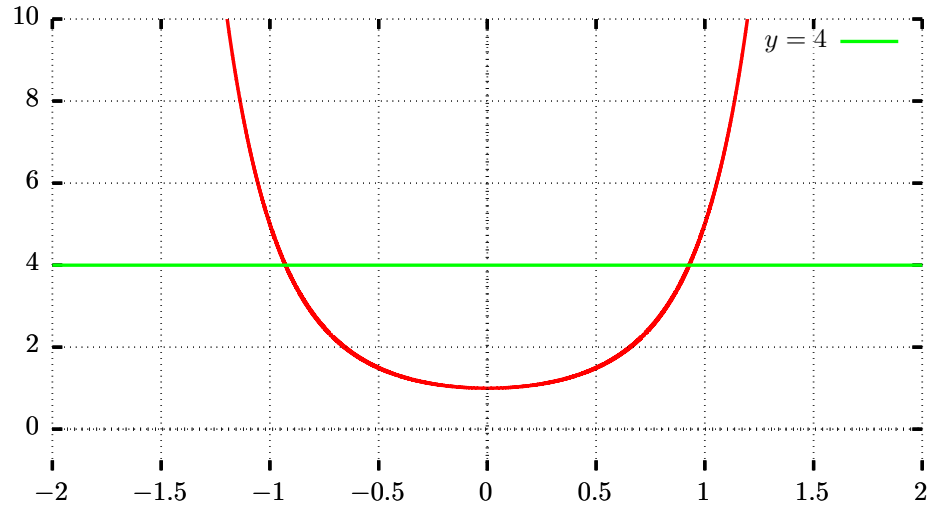
Injective

6.

$$F : \mathbb{R} \longrightarrow \mathbb{R}$$

$$x \mapsto F(x) = 5^{x^2}$$

GNU06: $y = 5^{x^2}$

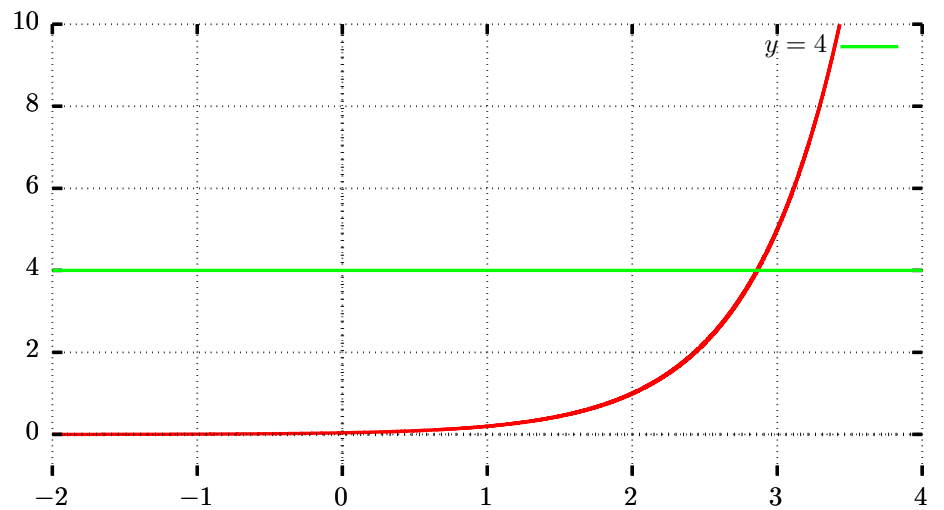


Injective

7.

$$\begin{aligned} F: \mathbb{R} &\longrightarrow \mathbb{R} \\ x &\mapsto F(x) = 5^{x-2} \end{aligned}$$

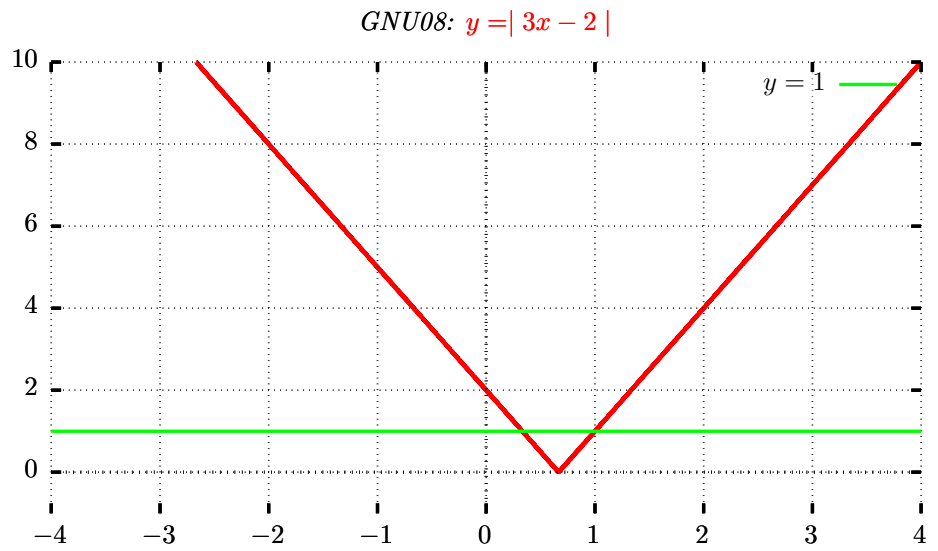
GNU07: $y = 5^{x-2}$



Injective

8.

$$\begin{aligned} F: \mathbb{R} &\longrightarrow \mathbb{R} \\ x &\mapsto F(x) = |3x - 2| \end{aligned}$$

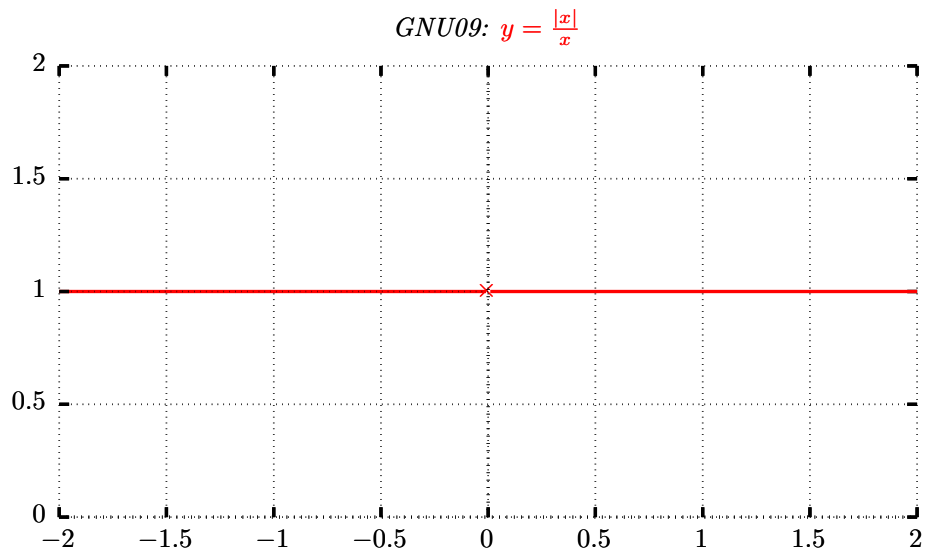


Not injective

9.

$$F : \mathbb{R} - \{0\} \longrightarrow \mathbb{R}$$

$$x \quad \mapsto \quad F(x) = \frac{|x|}{x}$$

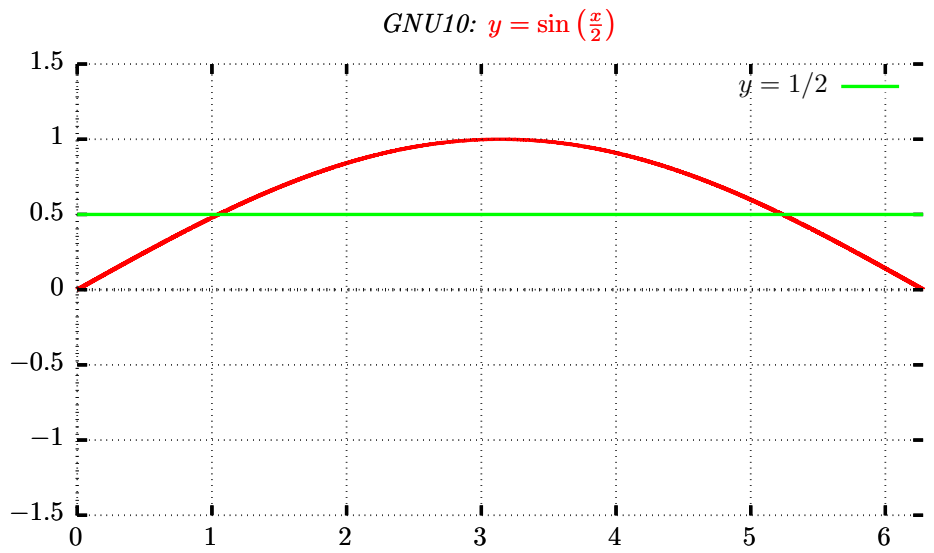


Not injective

10.

$$F : [0, 2\pi) \longrightarrow \mathbb{R}$$

$$x \quad \mapsto \quad F(x) = \sin\left(\frac{x}{2}\right)$$



Exercise 2. Determine if the following functions are injective.

1.

$$F : \mathbb{R} \longrightarrow \mathbb{R}$$

$$x \mapsto F(x) = x^4 + 3x^2 + 3$$

No, because $F(1) = F(-1)$

2.

$$F : \mathbb{N} \longrightarrow \mathbb{N}$$

$$n \mapsto F(n) = \sqrt{n}$$

*Yes, because the function $F : \mathbb{R} \longrightarrow \mathbb{R}$
 $x \mapsto F(x) = \sqrt{x}$ is injective*

3.

$$F : \mathbb{Z} \longrightarrow \mathbb{Z}$$

$$n \mapsto F(n) = \sqrt{3}n^2$$

No, because $F(-1) = F(1)$

4.

$$F : \mathbb{N} \longrightarrow \mathbb{N}$$

$$n \mapsto F(n) = \sin(n)$$

[Tricky]

5.

$$\begin{array}{rcl} F : \{1, 2, 3, 4, 5, 6\} & \longrightarrow & \{1, 2, 3, 4, 5, 6\} \\ & & 1 \mapsto 2 \\ & & 2 \mapsto 4 \\ & & 3 \mapsto 1 \\ & & 4 \mapsto 6 \\ & & 5 \mapsto 5 \\ & & 6 \mapsto 3 \end{array}$$

Injective

6.

$$\begin{array}{rcl} F : \{1, 2, 3, 4, 5, 6\} & \longrightarrow & \{1, 2, 3, 4, 5, 6\} \\ & & 1 \mapsto 2 \\ & & 2 \mapsto 4 \\ & & 3 \mapsto 1 \\ & & 4 \mapsto 2 \\ & & 5 \mapsto 5 \\ & & 6 \mapsto 3 \end{array}$$

Not injective

Exercise 3. Let A be a set with 10 elements and B a set with 9 elements. Is it possible for a function $f : A \rightarrow B$ to be injective? [No]

Exercise 4. Let A be a set with 10 elements and B a set with 11 elements. Is it possible for a function $f : A \rightarrow B$ to be injective? [Yes]

Exercise 5. Is it true that if $F_1, F_2 : \mathbb{R} \rightarrow \mathbb{R}$ are two functions of PERIOD $p_1, p_2 \in \mathbb{N}$ then the function

$$\begin{array}{rcl} H : \mathbb{R} & \longrightarrow & \mathbb{R} \\ x & \mapsto & F_1(x) + F_2(x) \end{array}$$

has a period $\text{lcm}(p_1, p_2)$? Motivate your answer. [Yes]

Exercise 6. Is it true that if $F_1, F_2 : \mathbb{R} \rightarrow \mathbb{R}$ are two functions of PERIOD $p_1, p_2 \in \mathbb{R}$ then the function

$$\begin{array}{rcl} H : \mathbb{R} & \longrightarrow & \mathbb{R} \\ x & \mapsto & F_1(x) + F_2(x) \end{array}$$

has PERIOD $\text{lcm}(p_1, p_2)$? Motivate your answer. [No]

Exercise 7. Is it true that if $F_1, F_2 : \mathbb{R} \rightarrow \mathbb{R}$ are two functions of PERIOD $p_1, p_2 \in \mathbb{Q}$ then the function

$$\begin{array}{rcl} H : \mathbb{R} & \longrightarrow & \mathbb{R} \\ x & \mapsto & F_1(x) + F_2(x) \end{array}$$

has PERIOD $\text{lcm}(p_1, p_2)$? Motivate your answer. [No]

Exercise 8. Determine the PERIOD of the function

$$\begin{aligned} F : \mathbb{R} &\longrightarrow \mathbb{R} \\ x &\mapsto \sin\left(\frac{1}{2}x\right) + \cos\left(\frac{1}{3}x\right) \end{aligned}$$

$[12\pi]$

Exercise 9. Determine the PERIOD of the function

$$\begin{aligned} F : \mathbb{R} &\longrightarrow \mathbb{R} \\ x &\mapsto \sin\left(\frac{1}{2}x\right) + \cos\left(\frac{1}{4}x\right) \end{aligned}$$

$[8\pi]$

Exercise 10. Determine the PERIOD of the function

$$\begin{aligned} F : \mathbb{R} &\longrightarrow \mathbb{R} \\ x &\mapsto \sin\left(\frac{1}{4}x\right) + \cos\left(\frac{1}{6}x\right) \end{aligned}$$

$[24\pi]$

Exercise 11. Determine the PERIOD of the function

$$\begin{aligned} F : \mathbb{R} &\longrightarrow \mathbb{R} \\ x &\mapsto \sin(2x) + \cos(3x) \end{aligned}$$

$[2\pi]$

Exercise 12. Determine the PERIOD of the function

$$\begin{aligned} F : \mathbb{R} &\longrightarrow \mathbb{R} \\ x &\mapsto \sin(4x) + \cos(6x) \end{aligned}$$

$[\pi]$

Exercise 13. Determine the PERIOD of the function

$$\begin{aligned} F : \mathbb{R} &\longrightarrow \mathbb{R} \\ x &\mapsto \sin(4x) + \cos(8x) \end{aligned}$$

$[\pi/2]$

Exercise 14. Determine the PERIOD of the function

$$\begin{aligned} F : \mathbb{R} &\longrightarrow \mathbb{R} \\ x &\mapsto \sin(\pi x) \end{aligned}$$

$[2]$

Exercise 15. Determine the PERIOD of the function

$$\begin{aligned} F : \mathbb{R} &\longrightarrow \mathbb{R} \\ x &\mapsto \sin(\sqrt{2}x) \end{aligned}$$

$[\sqrt{2}\pi]$

Exercise 16. *The function*

$$\begin{array}{rcl} F : \mathbb{R} & \longrightarrow & \mathbb{R} \\ x & \mapsto & \sin(\sqrt{2}x) + \cos(2x) \end{array}$$

has a period? Has a PERIOD? [NO]