

FCS  
Math: Functions  
Exercises

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**Exercise 1.** *Determine if the following functions are injective, drawing the graphs*

1.

$$\begin{array}{l} F : \mathbb{R} \longrightarrow \mathbb{R} \\ x \mapsto F(x) = x^2 \end{array}$$

2.

$$\begin{array}{l} F : \mathbb{R}_0^+ \longrightarrow \mathbb{R} \\ x \mapsto F(x) = x^2 \end{array}$$

3.

$$\begin{array}{l} F : \mathbb{R}_0^+ \longrightarrow \mathbb{R} \\ x \mapsto F(x) = \sqrt{x} \end{array}$$

4.

$$\begin{array}{l} F : [-1, +\infty) \longrightarrow \mathbb{R} \\ x \mapsto F(x) = \sqrt{x+1} \end{array}$$

5.

$$\begin{array}{l} F : [2, +\infty] \longrightarrow \mathbb{R} \\ x \mapsto F(x) = \log x - 2 \end{array}$$

6.

$$\begin{array}{l} F : \mathbb{R} \longrightarrow \mathbb{R} \\ x \mapsto F(x) = 5^{x^2} \end{array}$$

7.

$$\begin{array}{l} F : \mathbb{R} \longrightarrow \mathbb{R} \\ x \mapsto F(x) = 5^{x-2} \end{array}$$

8.

$$\begin{array}{l} F : \mathbb{R} \longrightarrow \mathbb{R} \\ x \mapsto F(x) = |3x - 2| \end{array}$$

9.

$$F: \mathbb{R} - \{0\} \longrightarrow \mathbb{R} \\ x \mapsto F(x) = \frac{|x|}{x}$$

10.

$$F: [0, 2\pi) \longrightarrow \mathbb{R} \\ x \mapsto F(x) = \sin\left(\frac{x}{2}\right)$$

**Exercise 2.** Determine if the following functions are injective.

1.

$$F: \mathbb{R} \longrightarrow \mathbb{R} \\ x \mapsto F(x) = x^4 + 3x^2 + 3$$

2.

$$F: \mathbb{N} \longrightarrow \mathbb{N} \\ n \mapsto F(n) = \sqrt{(n)}$$

3.

$$F: \mathbb{Z} \longrightarrow \mathbb{Z} \\ n \mapsto F(n) = \sqrt{(3)}n^2$$

4.

$$F: \mathbb{N} \longrightarrow \mathbb{N} \\ n \mapsto F(n) = \sin(n)$$

[Tricky]

5.

$$F: \{1, 2, 3, 4, 5, 6\} \longrightarrow \{1, 2, 3, 4, 5, 6\} \\ \begin{array}{ccc} 1 & \mapsto & 2 \\ 2 & \mapsto & 4 \\ 3 & \mapsto & 1 \\ 4 & \mapsto & 6 \\ 5 & \mapsto & 5 \\ 6 & \mapsto & 3 \end{array}$$

6.

$$F: \{1, 2, 3, 4, 5, 6\} \longrightarrow \{1, 2, 3, 4, 5, 6\} \\ \begin{array}{ccc} 1 & \mapsto & 2 \\ 2 & \mapsto & 4 \\ 3 & \mapsto & 1 \\ 4 & \mapsto & 2 \\ 5 & \mapsto & 5 \\ 6 & \mapsto & 3 \end{array}$$

**Exercise 3.** Let  $A$  be a set with 10 elements and  $B$  a set with 9 elements. Is it possible for a function  $f: A \longrightarrow B$  to be injective ?

**Exercise 4.** Let  $A$  be a set with 10 elements and  $B$  a set with 11 elements. Is it possible for a function  $f: A \longrightarrow B$  to be injective ?

**Exercise 5.** Is it true that if  $F_1, F_2 : \mathbb{R} \rightarrow \mathbb{R}$  are two functions of PERIOD  $p_1, p_2 \in \mathbb{N}$  then the function

$$\begin{aligned} H : \mathbb{R} &\longrightarrow \mathbb{R} \\ x &\mapsto F_1(x) + F_2(x) \end{aligned}$$

has a period  $\text{lcm}(p_1, p_2)$ ? Motivate your answer.

**Exercise 6.** Is it true that if  $F_1, F_2 : \mathbb{R} \rightarrow \mathbb{R}$  are two functions of PERIOD  $p_1, p_2 \in \mathbb{N}$  then the function

$$\begin{aligned} H : \mathbb{R} &\longrightarrow \mathbb{R} \\ x &\mapsto F_1(x) + F_2(x) \end{aligned}$$

has PERIOD  $\text{lcm}(p_1, p_2)$ ? Motivate your answer.

**Exercise 7.** Determine the PERIOD of the function

$$\begin{aligned} F : \mathbb{R} &\longrightarrow \mathbb{R} \\ x &\mapsto \sin\left(\frac{1}{2}x\right) + \cos\left(\frac{1}{3}x\right) \end{aligned}$$

**Exercise 8.** Determine the PERIOD of the function

$$\begin{aligned} F : \mathbb{R} &\longrightarrow \mathbb{R} \\ x &\mapsto \sin\left(\frac{1}{2}x\right) + \cos\left(\frac{1}{4}x\right) \end{aligned}$$

**Exercise 9.** Determine the PERIOD of the function

$$\begin{aligned} F : \mathbb{R} &\longrightarrow \mathbb{R} \\ x &\mapsto \sin\left(\frac{1}{4}x\right) + \cos\left(\frac{1}{6}x\right) \end{aligned}$$

**Exercise 10.** Determine the PERIOD of the function

$$\begin{aligned} F : \mathbb{R} &\longrightarrow \mathbb{R} \\ x &\mapsto \sin(2x) + \cos(3x) \end{aligned}$$

**Exercise 11.** Determine the PERIOD of the function

$$\begin{aligned} F : \mathbb{R} &\longrightarrow \mathbb{R} \\ x &\mapsto \sin(4x) + \cos(6x) \end{aligned}$$

**Exercise 12.** Determine the PERIOD of the function

$$\begin{aligned} F : \mathbb{R} &\longrightarrow \mathbb{R} \\ x &\mapsto \sin(4x) + \cos(8x) \end{aligned}$$

**Exercise 13.** Determine the PERIOD of the function

$$\begin{aligned} F : \mathbb{R} &\longrightarrow \mathbb{R} \\ x &\mapsto \sin(\pi x) \end{aligned}$$

**Exercise 14.** Determine the *PERIOD* of the function

$$\begin{aligned} F : \mathbb{R} &\longrightarrow \mathbb{R} \\ x &\mapsto \sin(\sqrt{2}x) \end{aligned}$$

**Exercise 15.** The function

$$\begin{aligned} F : \mathbb{R} &\longrightarrow \mathbb{R} \\ x &\mapsto \sin(\sqrt{2}x) + \cos(2x) \end{aligned}$$

has a period? Has a *PERIOD*?