FCS
Math: Functions
Lesson

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Definition 1. *Given* A, B *sets,*

- $A \subseteq B \iff$ *for all* $a \in A$ *we have* $a \in B$*.*
- $A = B \iff A \subseteq B$ *and* $A \supseteq B$*.*

It is easy to see that

- $A \not\subseteq B \iff$ *there is* $a \in A,\ a \notin B$*.*
- $A \neq B \iff ($ *there is* $a \in A,\ a \notin B)$ *or* $($ *there is* $b \in B,\ a \notin A)$*.*

The previous definition using $\forall$ for "for all" and $\exists$ for "there is":

Definition 2. *Given* A, B *sets,*

- $A \subseteq B \iff \forall\ a \in A$ *we have* $a \in B$*.*
- $A = B \iff A \subseteq B$ *and* $A \supseteq B$*.*

It is easy to see that

- $A \not\subseteq B \iff \exists\ a \in A,\ a \notin B$*.*
- $A \neq B \iff (\exists\ a \in A,\ a \notin B)$ *or* $(\exists\ b \in B,\ a \notin A)$*.*

Definition 3. *Given* A, B *subsets of a set* U *we have that*

- $A \cup B = \{c \in U \ \mid\ c \in A \text{ or } c \in B\}$
- $A \cap B = \{c \in U \ \mid\ c \in A \text{ and } c \in B\}$
- $A - B = \{c \in U \ \mid\ c \in A \text{ and } c \notin B\}$

Definition 4. *Given* A, B *sets* $A \times B = \{(a, b) \ \mid\ a \in A \text{ and } b \in B\}$ *is the* cartesian product *of* A, B*.*

Definition 5. *For a function*

$$F: \begin{array}{ccc} A & \longrightarrow & B \\ a & \mapsto & F(a) \end{array},$$

we say that A is the domain *of F, B is the* codomain *of F and $F(a)$ is the* formula *of F.*

Definition 6. *Given the functions*

$$F: \begin{array}{ccc} A & \longrightarrow & B \\ a & \mapsto & F(a) \end{array}, \qquad G: \begin{array}{ccc} C & \longrightarrow & D \\ c & \mapsto & G(c) \end{array}$$

we have

$$F \equiv G$$

if and only if

$$A = C,\ B = D,\ \ \forall\ a \in A,\ F(a) = G(a)$$

we say that F, G are equal as functions*.*

Definition 7. *Given a function*

$$F: \begin{array}{ccc} A & \longrightarrow & B \\ a & \mapsto & F(a) \end{array}$$

$$GR(f) = \{(a, F(a)) \ \mid \ a \in A\} \subset A \times B$$

is the graph of F