$A = \mathbb{N}, \mathbb{Z} = \{-3, -2, 0, 3, \ldots, \ldots\}$

$\mathbb{N} = 0, 1, 2, \ldots$

$\mathbb{Q} = 1/2, 3/9, 1 \ldots$

$\mathbb{R} = 0, 1/2, \sqrt{2}, \sqrt[3]{5}, \pi, \ldots$

$(\mathbb{C} = 1 + 3i \ \ldots)$

A SET $= \{1, 2, 3, 5\}$

$A = \{1, 2, 3, 5, \sqrt{2}, 3\}$

$A = \{x \in \mathbb{R} \mid x^2 = 1\} = \{\pm 1\}$

$A = \{a^2 \mid a \in \mathbb{N}\} = \{0^2, 1^2, 2^2, \ldots\}$

$\{0, 1, 4, 9, \ldots\}$

$A = \{x \in \mathbb{R} \mid x^2 = 1\} = \{a \in \mathbb{R} \mid a^2 = 1\} = \{\square \in \mathbb{R} \mid \square^2 = 1\}$

$= \{\triangle \in \mathbb{R} \mid \triangle^2 = 1\}$

A, B SETS

$3 \in A \quad 3 \notin B$

$A \subseteq B$ $\qquad$ $A = \{1, 2, 3\} \nsubseteq \{1, 2\} = B$

$A = B$ $\qquad$ $A = \{1, 2, 3\} \supseteq \{1, 2\} = B$

$A \neq B$

$A \subseteq B \Leftrightarrow \forall a \in A \ a \in B$

$A \nsubseteq B \qquad \exists a \in A \ a \notin B$

$A = B \qquad \forall a \in A \ a \in B$ AND $\forall b \in B \ b \in A$

$A \subseteq B$ AND $A \supseteq B$

$A = \{x \in \mathbb{R} \mid x^2 = 1\}$ $\qquad$ $B = \{-1, +1\}$ $\qquad$ $A = B$

$A = \{a \in \mathbb{R} \mid 3a - 1 = 4\}$ $\qquad$ $B = \{a \in \mathbb{R} \mid 3a - 5 = 0\}$

$=$

$A = \{x \in \mathbb{R} \mid x^2 - 3x + 2 = 0\}$ $\qquad$ $B = \{1, 2\} \subseteq \mathbb{R}$

$A = B$ by 2-DEG. EQ. FORMULA

$A = \{x \in \mathbb{R} \mid \underline{x^7 + x^3 + x + 3 = 0}\}$ $\qquad$ $B = \{-1, 1\}$

$A = B$?

$-1 \in A \qquad -1^7 + (-1)^3 + (-1) + 3 = 0$ $\qquad$ OK

$1 \in A \qquad 1^7 + 1^3 + 1 + 3 = 0$ $\qquad$ NOT TRUE

$A \neq B$

$\boxed{A = C\ ?}$ $\qquad$ $-1 \in A \Rightarrow A \supseteq C$ OK $\qquad$ $C = \{-1\}$

$A = \{x \in \mathbb{N} \mid x \text{ IS EVEN}\} = \{0, 2, 4, 6, \ldots\} = B$

$= \{2n \mid n \in \mathbb{N}\} = C$

$\# A = \infty$ $\quad\quad \# \{1, 2, 3, 4\} = 4$

$A \subseteq \mathbb{N} \quad\quad A = B = C$

$A = \{x \in \mathbb{N} \mid x \text{ IS A MULTIPLE OF } 6\} = \{0, 6, 12, 18, 24, \ldots\}$

$B = \{a \in \mathbb{N} \mid a \text{ IS A MULTIPLE OF } 4\} = \{0, 4, 8, 12, 16, \ldots\}$

$A \cap B = \{c \in \mathbb{N} \mid c \in A \text{ AND } c \in B\}$

$= \{0, 12, 24, 36, \ldots\}$

$= \{y \in \mathbb{N} \mid y \text{ IS A MULTIPLE OF } \underbrace{12}_{\text{LCM}(4,6)}\}$

$A \cup B = \{c \in \mathbb{N} \mid c \in A \text{ OR } c \in B\}$

$= \{0, 4, 6, 8, 12, 16, 18, \ldots\}$

$= \{y \in \mathbb{N} \mid y \text{ IS A MULTIPLE OF } ?\}$

$A - B = \{a \in A \mid a \notin B\} = \{0, 6, \cancel{12}, 18, \cancel{24}, \ldots\}$

$= \{0, 6, 18, \ldots\}$

$= \{y \in \mathbb{N} \mid y \text{ IS A MULTIPLE OF } ?\}$

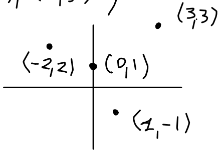
$A, B \subseteq \underbrace{U}_{\text{UNIVERSE}}$ $\quad$ IN THIS CASE : $U = \mathbb{N}$

$\boxed{A \times B}$ CARTESIAN PRODUCT A, B SETS

$$= \{(a,b) \mid a \in A \;\; b \in B\}$$ SET OF THE PAIRS.

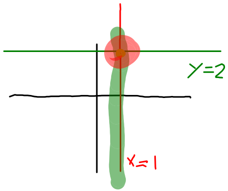
Ex $\{1,2\} \times \{0,4,9\} = \{(1,0), (1,4), (1,9), (2,0), (2,4), (2,9)\}$

Ex REAL PLANE: $\mathbb{R} \times \mathbb{R} = \mathbb{R}^2$



$$\{1\} \times \mathbb{R} = \boxed{\{(1,x) \mid x \in \mathbb{R}\}} = A$$

$$= \{(x,y) \in \mathbb{R}^2 \mid x = 1\}$$



$$\mathbb{R} \times \{2\} = \{(x,2) \mid x \in \mathbb{R}\}$$

$$= \{(x,y) \in \mathbb{R}^2 \mid y = 2\} = B$$

$A \cap B = \{(1,2)\}$ $\quad$ $A \cup B =$ "THE CROSS" ← WRITE IT DOWN

$A - B = \boxed{\{(1,y) \in \mathbb{R}^2 \mid y \in \mathbb{R}\}} - B =$ "THE LINE WITHOUT A POINT"

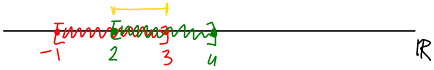
$[1,2] = \{x \in \mathbb{R} \mid 1 \leq x \leq 2\}$

$(-3,0) = \{a \in \mathbb{R} \mid -3 < a < 0\} = \cancel{]-3,0[}$

$(-\infty,-1) = \{A \in \mathbb{R} \mid A < -1\}$

$(a,b) \equiv \cancel{]a,b[}$

$[-1,3] \cap [2,4] = [2,3]$



$[-1,3] \cup [2,4] = [-1,4]$

LIMITED

$A \subseteq \mathbb{R}$ $A = [1,2]$

2 is the sup of A

2 is ~~a~~ the maximum of A

$B \subseteq \mathbb{R}$ $B = [1,2)$

2 is not ~~a~~ the maximum of B

2 is ~~a~~ the sup of B

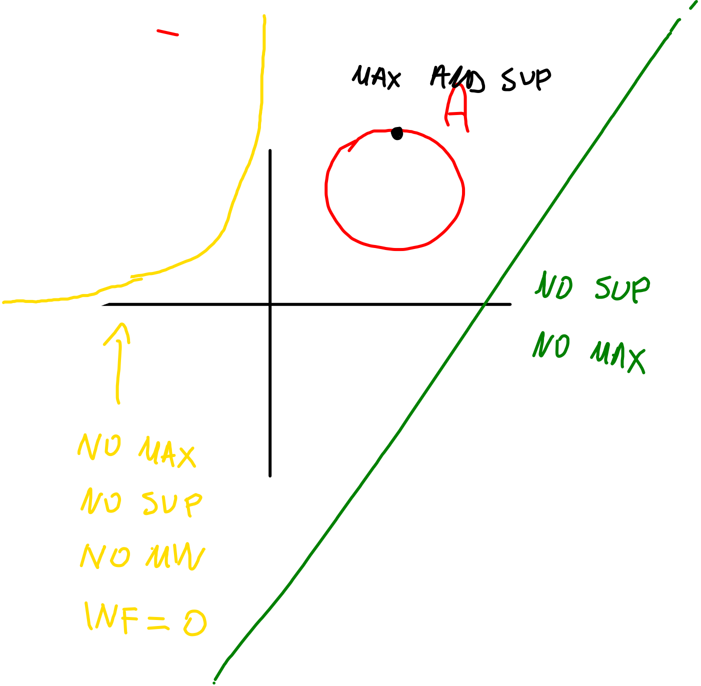
THERE IS ALWAYS A SUP FOR $A \subseteq \mathbb{R}$?

$[1,+\infty)$ HAS A SUP? NO

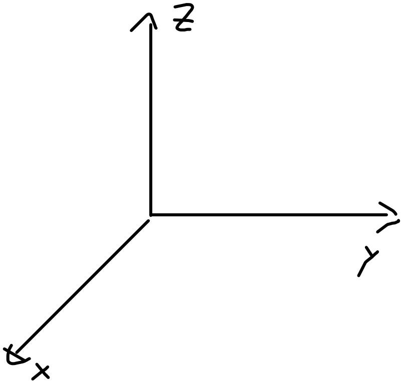
UNLIMITED

MAX $\leftrightarrow$ MIN

SUP $\leftrightarrow$ INF



A HAS MAX
SUP W.R.T. HOZIN.



$$\mathbb{R}^5 = \{ (a, b, c, d, e) \mid a, b, c, d, e \in \mathbb{R} \}$$

FUNCTIONS

DOMAIN $\quad$ CODOMAIN

F FUNCTION $\qquad F : A \longrightarrow B$

A, B SETS $\qquad a \rightsquigarrow \boxed{F(a)}$: THIS IS A FORMULA

A FORMULA

$$F : \mathbb{R} \longrightarrow \mathbb{R}$$
$$x \rightsquigarrow F(x) = \underbrace{3x^2 - 1}_{\text{FORMULA}}$$

$F : \mathbb{R} \to \mathbb{R}$ $\qquad G : \mathbb{R} \to \mathbb{R}$

$x \rightsquigarrow \sin(x)$ $\qquad x \rightsquigarrow \tan(x)$

$H : \mathbb{N} \to \mathbb{N}$ $\qquad H(1) = 3 \cdot 1^2 - 1 = 3 - 1 = 2$

$x \rightsquigarrow 3x^2 - 1$ $\qquad H(3) = 3 \cdot 3^2 - 1 = 3 \cdot 9 - 1 = 26$

$a \in \mathbb{N}$ $\qquad H(a+1) = 3 \cdot (a+1)^2 - 1$

$$= 3(a^2 + 2a + 1) - 1$$
$$= 3a^2 + 6a + 2$$

$\mathbb{N}$? $\qquad \square \in \mathbb{N}$ $\qquad H(\square) = 3 \cdot \square^2 - 1$

$H(1/2) = ?$ DOES NOT EXIST

$\bar{H} : \mathbb{Q} \to \mathbb{Q}$ $\qquad \bar{H}(1/2) = 3\left(\frac{1}{2}\right)^2 - 1$ OK

$x \rightsquigarrow 3x^2 - 1$ $\qquad \bar{H} \neq H$

<u>Pb</u> $\quad F : A \to B$ $\qquad H : C \to D$

$a \rightsquigarrow F(a)$ $\qquad c \rightsquigarrow H(c)$

$F \equiv H \iff$ $A = C$, $B = D$

("THE FORMULAS HAVE TO BE THE SAME"

OR

$\forall a \in A \quad F(a) = H(a)$

$F : \mathbb{R} \to \mathbb{R}$ $\qquad G : \mathbb{R} \to \mathbb{R}$

$x \rightsquigarrow 1$ $\qquad x \rightsquigarrow \sin^2(x) + \cos^2(x)$

$\forall x \in \mathbb{R} \quad 1 = \sin^2(x) + \cos^2(x) \quad \Rightarrow \quad F \equiv G$

WHAT ABOUT $\sin(2)$? $\qquad \sin^2(2) + \cos^2(2) = 1$

A : NATURAL NUMBERS ON A COMPUTER

$\bar{x}$ IS THE MAXIMUM NUMBER YOU CAN REPRESENT ON THE COMPUTER

$f : A \to A$ $\qquad g : A \to A$

$n \rightsquigarrow n + 1$ $\qquad n \rightsquigarrow n + \bar{x} - \bar{x} + 1$

$$\qquad\qquad\qquad\qquad\qquad \| $$
$$\qquad\qquad\qquad\qquad\qquad n+1$$

$$f \equiv g$$

$F : \mathbb{N} \to \mathbb{R}$

$n \rightsquigarrow \sqrt{n}$

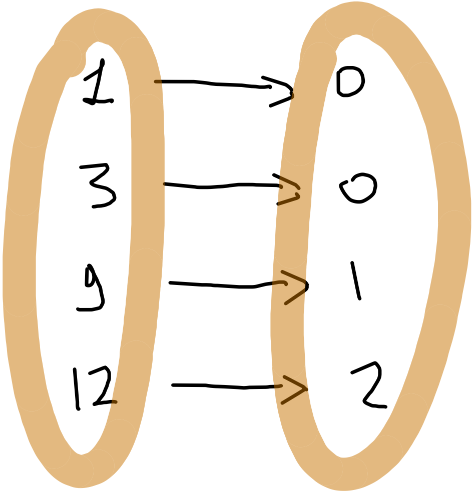
$F(9) = 3$

$F(2) = \sqrt{2}$

$\vdots$

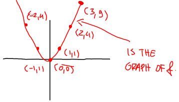
CASE I: DOMAIN, CODOMAIN ARE FINITE LISTS

$$f : \{1, 3, 9, 12\} \longrightarrow \{0, 1, 2\}$$



$$f(9) = 1$$

$$f(12) = 2$$



$f : \mathbb{R} \to \mathbb{R}$

$x \rightsquigarrow x^2 = f(x)$

$f(0) = 0 \iff (0,0) \in GR(f)$

$f(1) = 1 \iff (1,1) \in GR(f)$

$f(2) = 4 \iff (2,4) \in GR(f)$

$f(3) = 9 \iff (3,9) \in GR(f)$

$\vdots$

$GR(f) = \{ (x,y) \in \mathbb{R}^2 \mid \underset{\substack{\| \\ f(x)}}{y} = x^2 \}$

$= \{ (x, f(x)) \mid x \in \mathbb{R} \} = \{ (x, x^2) \mid x \in \mathbb{R} \}$

$F : A \to B$

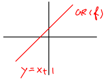
$a \rightsquigarrow F(a)$

$GR(F) = \{ (\underset{A}{a}, \underset{B}{F(a)}) \mid a \in A \} \subseteq A \times B$

$f : \mathbb{R} \to \mathbb{R}$

$x \rightsquigarrow x+1$

$f(x) = x+1$



$\boxed{g : \mathbb{R} \to \mathbb{R} \quad x \rightsquigarrow 1/x}$ IS A FUNCTION ? NO

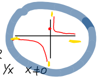
$g(3) = 1/3 \quad g(1) = 1 \quad \nexists g(0)$ DOES NOT EXIST

$g(2/3) = 3/2$

I^{st} CASE

$g' : \mathbb{R} \to \mathbb{R}$

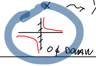
$x \rightsquigarrow \begin{cases} 1/x & x \neq 0 \\ 3 & x = 0 \end{cases}$



II^{nd} CASE

$g'' : \mathbb{R} - \{0\} \to \mathbb{R}$

$x \rightsquigarrow 1/x$

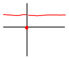


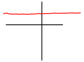
$f : \mathbb{R} \to \mathbb{R}$

$x \rightsquigarrow \begin{cases} x/x & x \neq 0 \\ 0 & x = 0 \end{cases}$

$g : \mathbb{R} \to \mathbb{R}$

$x \rightsquigarrow 1$





$f \equiv g$? NO

DOMAIN, CODOMAIN : SAME

$\exists$ (THERE IS) $0 \in \mathbb{R}$ $f(0) = 0 \neq 1 = g(0)$

GRAPHS WE KNOW $\quad f:\mathbb{R}\to\mathbb{R}$

$f(x)=ax+b \quad a,b\in\mathbb{R}$ LINES

$f(x)=ax^2+bx+c \quad a,b,c\in\mathbb{R}$ — WE KNOW

$f(x)=1/x \quad : \quad f: \mathbb{R}-\{0\}\to\mathbb{R}$

$f(x)=\sin(x),\ \cos(x),\ \tan(x)$ — WE WILL DETAIL

$f(x)=3^x,\ (1/2)^x,\ \log_3 x$

$f(x)=\sqrt{x}$

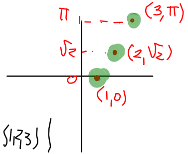
$f:\{1,2,3\}\to\mathbb{R}$

$1 \rightsquigarrow 0$

$2 \rightsquigarrow \sqrt{2}$

$3 \to \pi$

$GR(f) = \{(x,f(x)) \mid x\in\{1,2,3\}\} = \{(1,0),(2,\sqrt{2}),(3,\pi)\}$



Pb $\quad f:\mathbb{R}\to\mathbb{R},\ x\rightsquigarrow f(x) \longmapsto GR(f)$

$\longleftarrow$

FROM "THE GRAPH" TO THE FUNCTION

$D \subseteq \mathbb{R}\times\mathbb{R}$