FCS
Math: Functions
Exercises

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February 25, 2021

Exercise 1. *Given the sets*

$$A = \{1, 3, 5, 7, 14\} \text{ and } B = \{-2, 3, 4, 8\}$$

Describe $A \cup B$, $A \cap B$, $A - B$, $A \times B$.

Exercise 2. *Given the sets*

$A = \{a \in \mathbb{N} \mid a \text{ is a multiple of } 12\}$ *and* $B = \{k \in \mathbb{N} \mid k \text{ is a multiple of } 15\}$

Describe $A \cup B$, $A \cap B$, $A - B$, $A \times B$.

Exercise 3. *Given the sets*

$$A = \{(a, a^2) \in \mathbb{R}\} \text{ and } A = \{(b, b) \in \mathbb{R}\}$$

Describe $A \cup B$, $A \cap B$, $A - B$, $A \times B$.

Exercise 4. *Draw on the* $\mathbb{R}^2$ *plane the sets*

$$[1, 2] \times [1, 1],\ [-1, 2] \times [2, +\infty],\ \{(1, 1), (2, 3), (3, 7)\}$$

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Exercise 5. *Given the sets*

$$\begin{aligned} A &= \{a \in \mathbb{R} \mid x^3 - 4x^2 + x + 6 = 0 \in \mathbb{R}\} \\ B &= \{a \in \mathbb{R} \mid x^4 - 5x^2 + 4 = 0 \in \mathbb{R}\} \\ C &= \{a \in \mathbb{R} \mid x^4 - 2x^3 + 4x^2 - 6x + 3 = 0 \in \mathbb{R}\} \end{aligned}$$

Detail the equalities and inclusions between A, B, C

Exercise 6. *We have the function*

$$\begin{array}{rccc} F: & \mathbb{R} & \longrightarrow & \mathbb{R} \\ & x & \mapsto & 3x^2 - x + 1 \end{array}$$

Compute

1. $F(1) = F(-1)$*?*
2. $F(1), F(0), F(5)$.
3. *Given* $a \in \mathbb{R}$*, compute* $F(a-1), F(3a^2-2), F(\sqrt{a^2+1})$.
4. *Given* $\clubsuit \in \mathbb{R}$*, with* $\clubsuit > 0$*, compute* $F(\clubsuit - 2)$, $F(2\clubsuit)$, $F(\sqrt{\clubsuit})$.

Exercise 7. *We have the function*

$$\begin{array}{rccc} F: & \mathbb{R} & \longrightarrow & \mathbb{R} \\ & x & \mapsto & x^2 + 1 \end{array}$$

Compute

1. $F(1) = F(-1)$*?*
2. *For which* $a \in \mathbb{R}$ *we have* $F(a) = F(-a)$.
3. *For which* $y \in \mathbb{R}$ *we have* $F(y) = F(y+1)$.
4. *For which* $b \in \mathbb{R}$ *we have* $F(b+2) = F(2b+3)$.

Exercise 8. *We have the functions*

$$\begin{array}{rccc} F: & \mathbb{R} & \longrightarrow & \mathbb{R} \\ & x & \mapsto & x^2 \end{array}, \qquad \begin{array}{rccc} G: & \mathbb{R} & \longrightarrow & \mathbb{R} \\ & x & \mapsto & x \end{array}$$

1. *Is it true that* $F \equiv G$*?*
2. *Is there an* $a \in \mathbb{R}$ *such that* $F(a) = G(a)$*?*

Exercise 9. *We try to describe a function by*

$$\begin{array}{rccc} F: & \mathbb{Q} & \longrightarrow & \mathbb{Q} \\ & p/q & \mapsto & p^2/q^2 \end{array}$$

Is F *a well defined function?*

Exercise 10. *We try to describe a function by*

$$\begin{array}{rccc} F: & \mathbb{Q} & \longrightarrow & \mathbb{Q} \\ & p/q & \mapsto & p+q \end{array}$$

Is F *a well defined function? If not, how can we modify the formula of* F *to have a well defined function?*

Exercise 11. *In the plane* $\mathbb{R}^2$*, draw the graphs of the functions*

$$\begin{array}{rccc} F: & \mathbb{R} & \longrightarrow & \mathbb{R} \\ & x & \mapsto & x^2 \end{array}, \quad \begin{array}{rccc} F: & \mathbb{R} & \longrightarrow & \mathbb{R} \\ & b & \mapsto & b^2 - 1 \end{array}, \quad \begin{array}{rccc} F: & \mathbb{R} & \longrightarrow & \mathbb{R} \\ & c & \mapsto & 3c^2 \end{array}, \quad \begin{array}{rccc} F: & \mathbb{R} & \longrightarrow & \mathbb{R} \\ & \heartsuit & \mapsto & \heartsuit^2 + 3 \end{array}$$