

FCS  
Math: Functions  
Exercises

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Some solutions will be discussed in class

1. Prove that  $|\mathbb{N}| = |\mathbb{Q}|$  using the Cantor-Schröder-Bernstein theorem.  
[This is quite easy using the right idea].
2. Prove that  $|[0, 1]| = |\mathbb{R}|$  using the Cantor-Schröder-Bernstein theorem.
3. Prove that  $|[0, 1]| = |\mathbb{R}|$  by finding an one-to-one correspondence.  
[Difficult]
4. Prove that  $|\mathbb{N}| = |\mathbb{Q}|$  directly by building a function  $F : \mathbb{N} \rightarrow \mathbb{Q}$ .
5. Find an infinite set whose cardinality is bigger than  $|\mathbb{R}|$ .
6. Prove that  $|\mathbb{N}| = |\mathbb{Z} \times \mathbb{Z}|$ .
7. Prove that  $|\mathbb{N}^3| = |\mathbb{N}|$ .
8. Prove that  $|\mathbb{Q}^2| = |\mathbb{N}|$ .
9. Prove that  $\forall n \in \mathbb{N}, |\mathbb{N}^n| = |\mathbb{N}|$ .
10. Prove that  $\forall n \in \mathbb{N}, |\mathbb{Q}^n| = |\mathbb{N}|$ .
11. Let  $\mathbb{Z}[x] = \{\text{polynomials with coefficients in } \mathbb{Z}\}$ .  
Prove that  $|\mathbb{Z}[x]| = |\mathbb{N}|$ . [Difficult]
12. Let  $\mathbb{Q}[x] = \{\text{polynomials with coefficients in } \mathbb{Q}\}$ .  
Prove that  $|\mathbb{Q}[x]| = |\mathbb{N}|$ . [Difficult]
13. Propose as many sets as possible whose cardinality is bigger than the cardinality of  $\mathbb{R}$ .