FCS
Math: Functions

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We remind the definition of continuous function

Remark 1. *We remember that a* continuous *function* $F : A \longrightarrow B$*, with* $A, B \subseteq \mathbb{R}$ *is a function whose graph can be drawn without raising the pen from the paper.*

Proposition 1. *Let* $F : A \longrightarrow B$ *be a continuous, increasing function and* $[a, b] \subset \mathbb{R}$*. Then*

$$F([a, b]) = [F(a), F(b)]$$

Proposition 2. *Let* $F : A \longrightarrow B$ *be a continuous, decreasing function and* $[a, b] \subset \mathbb{R}$*. Then*

$$F([a, b]) = [F(b), F(a)]$$

Proposition 3. *A monotone function* $F : A \longrightarrow B$ *is injective.*

Proposition 4. *If* $F : A \longrightarrow B$ *is an invertible, increasing (decreasing) function, then its inverse is also increasing (decreasing).*

Proposition 5. *If* $F : A \longrightarrow B$ *is a function, and* $A' \cup A'' \subset A$*,* $B' \cup B'' \subset B$ *then*

$$F(A' \cup A'') = F(A') \cup F(A'') \text{ and } F^{-1}(B' \cup B'') = F^{-1}(B') \cup F^{-1}(B'')$$

Remark that $F(\cdot)$ *is the image and* $F^{-1}(\cdot)$ *is the counterimage, NOT NECESSARILY THE INVERSE. The function* F *is not necessarily invertible.*

Definition 1. *A function like*

$$\begin{array}{rccc} F : & \mathbb{R} & \longrightarrow & \mathbb{R} \\ & x & \mapsto & \begin{cases} x^2 - 2 & x \leq 0 \\ -x + 1 & x > 0 \end{cases} \end{array}$$

is a picecewise *function. and the two functions*

$$\begin{array}{rccc} F_{|(-\infty,0]} : & (-\infty, 0] & \longrightarrow & \mathbb{R} \\ & x & \mapsto & x^2 - 2 \end{array} \text{ and } \begin{array}{rccc} F_{|(0,+\infty)} : & (0, +\infty) & \longrightarrow & \mathbb{R} \\ & x & \mapsto & -x + 1 \end{array}$$

are its pieces.