Ex

$\sqrt{2x-5} - 2x + 2 < 0$

$\sqrt{2x-5} < 2x - 2$

$\boxed{2x-5 \geq 0 \quad E.F.}$

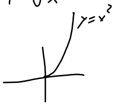
$F: \mathbb{R}_0^+ \longrightarrow \mathbb{R}_0^+ \quad \uparrow$

$x \rightsquigarrow x^2$

$\begin{cases} 2x-5 \geq 0 \\ 2x-2 \geq 0 \\ (\sqrt{2x-5})^2 < (2x-2)^2 \end{cases}$

$2x-5 \geq 0$ INVERSE OF $\sqrt{x}$

IF $2x-2 < 0$



$\begin{cases} x \geq 5/2 \\ x \geq 1 \\ 2x-5 < 4x^2-8x+4 \end{cases} \Rightarrow x \geq 5/2$

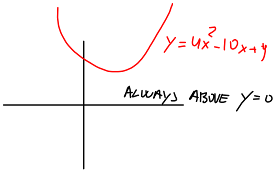
$\cup \begin{cases} 2x-5 \geq 0 \\ 2x-2 < 0 \\ \sqrt{2x-5} < 2x-2 \end{cases}$

$\emptyset$ $\nexists$ SOL

$\begin{cases} x \geq 5/2 \\ 4x^2-10x+9 > 0 \iff \forall x \end{cases}$

$x_{1,2} = \dfrac{10 \pm \sqrt{100-16\cdot 9}}{8}$

$\nexists$ ROOTS



$\boxed{x \geq 5/2}$ SOLUTIONS OF THE INEQUALITY

Ex

$$\sqrt{x-3} > x$$

$x-3 \geq 0$ E.F.

$F: \mathbb{R}_0^+ \to \mathbb{R}_0^+ \uparrow$

$x \mapsto x^2$

$$\begin{cases} x-3 \geq 0 \\ x \geq 0 \\ \left(\sqrt{x-3}\right)^2 > x^2 \end{cases} \cup \begin{cases} x-3 \geq 0 & x \geq 3 \\ x < 0 \mapsto & x < 0 \\ \sqrt{x-3} > x & \forall x \end{cases}$$

$+ \quad -$

NO SOL

$\Downarrow$

$\boxed{\text{NO SOL}}$

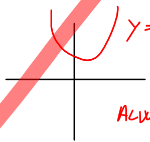
$$\begin{cases} x \geq 3 \\ x \geq 0 \\ x-3 > x^2 \end{cases} \iff x \geq 3$$

$$\begin{cases} x \geq 3 \\ x^2 - x + 3 < 0 \quad \forall x \end{cases}$$

$$x_{1,2} = \frac{1 \pm \sqrt{1-12}}{2}$$

NO ROOTS $\to$ SOL. TO THE INEQUALITY

$x \geq 3$



$y = x^2 - x + 3$

ALWAYS ABOVE $y = 0$

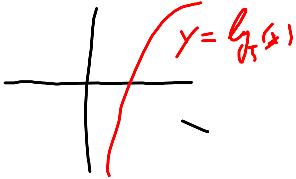
$$\{\, x \geq 3$$

$$\left\{ 3x^2 - 2x - 2 > 0 \right.$$

$$\Downarrow$$

$$\begin{cases} x \geq 3 \\ 3x^2 - 2x - 2 > 0 \end{cases} \cup \begin{cases} x < 3 \\ 3x^2 - 2x - 2 \end{cases}$$

Ex • $\log_5(x^2+1) > 3x-2$



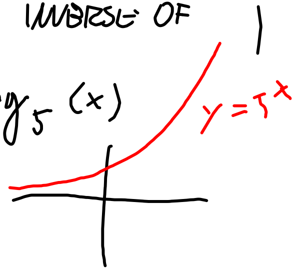
$$\Updownarrow$$

$$5^{\log_5(x^2+1)} > 5^{3x-2}$$

$F: \mathbb{R} \to \mathbb{R}^+$

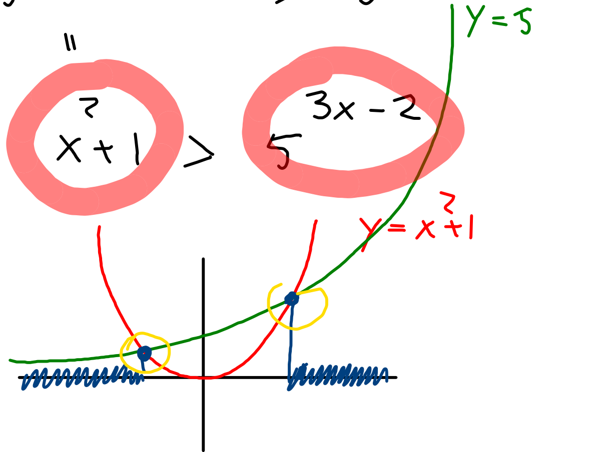
$x \mapsto 5^x$

THE INVERSE OF $\log_5(x)$



$\|$

• $x^2+1 > 5^{3x-2}$



Ex — $5^{x^2+1} < 5^{3x-2}$

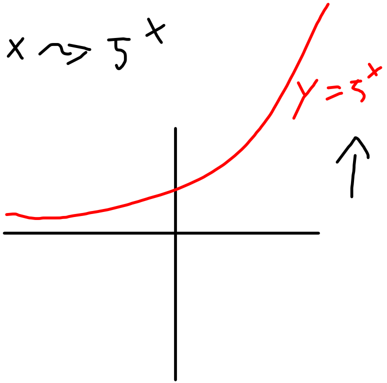
$F : \mathbb{R} \rightarrow \mathbb{R}$

$x \rightsquigarrow 5^x$

$F(x^2+1) < F(3x-2)$

$\Downarrow$ OK, $\uparrow$

$x^2+1 < 3x-2$



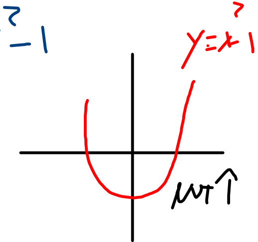
$H(x^2+1) < H(3x-2)$

$H : \mathbb{R} \rightarrow \mathbb{R}$

$\Downarrow$? NOT OK, NOT $\uparrow$

$x \rightsquigarrow x^2-1$

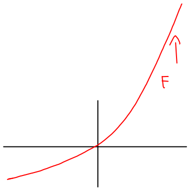
$x^2+1 < 3x-2$



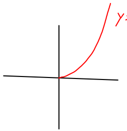
Pb: IS AN ↑ FUNCTION INJECTIVE ?

$\Updownarrow$

ANY HORIZ. LINE THAT INTERSECTS INTERSECTS ONCE ONLY

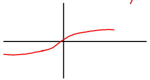


YES



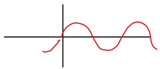
$y=x^2$

$y=\arctan(x)$ ↑ INJ



$F: \mathbb{R}_0^+ \to \mathbb{R}_0^+$

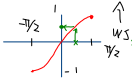
$x \mapsto x^2$



$y=\sin(x)$

$F: [-\pi/2, \pi/2] \to [-1,1]$

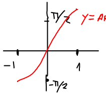
$x \mapsto \sin(x)$



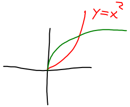
↑ INJ, SURJ

F IS INVERTIBLE

ARCSIN $= F^{-1}: [-1,1] \to [-\pi/2, \pi/2]$



$y=\arcsin(x)$



$y=x^2$ ↑

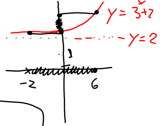
$y=\sqrt{x}$

PROP: IF $F: A \to B$ IS INVERTIBLE AND ↑ (↓)

F^{-1} IS ↑ (↓)

Ex $F: \mathbb{R} \to \mathbb{R}$ $\uparrow$

$x \rightsquigarrow 3^x + 2$



$$\boxed{F([-2,6]) = [F(-2), F(6)]}$$

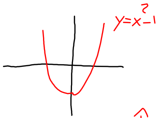
$$= [3^{-2}+2,\ 3^6+2]$$

$$= \left[\frac{1}{9}+2,\ 3^6+2\right]$$

PROPOSITION IF $F: A \to B$ IS $\uparrow$

$$F([a,b]) = [F(a), F(b)]$$

NOT ALWAYS THE CASE



$$F([0,2]) \stackrel{?}{=} [F(0), F(2)]$$

$$= [-1, 3]$$

TRUE

NOT $\uparrow$

$F: \mathbb{R} \to \mathbb{R}$ NOT $\uparrow$

$$F([1/2, 2]) \stackrel{?}{=} [F(1/2), F(2)]$$

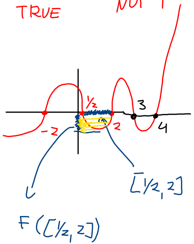
$\|$

$\|$

$[0, 0]$

$\|$

0

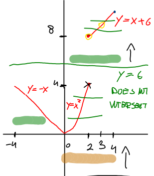


PROPOSITION $F: A \to B$ $\downarrow$

$$F([a,b]) = [F(b), F(a)]$$

$F : [-4,4] \longrightarrow \mathbb{R}_0^+$

$$x \rightsquigarrow \begin{cases} -x & -4 \leq x \leq 0 \\ x^2 & 0 < x < 2 \\ x+6 & 2 \leq x \leq 4 \end{cases}$$



NOT CONTINUOUS

$F\big|_{[0,4]}$ $\uparrow$ IS INVERTIBLE? NOT INVERTIBLE

IS INJ BECAUSE EVERY LINE THAT INTERSECTS DOES THAT ONLY ONCE

$$F([0,3])$$

1st P. $\quad$ 2nd P.

$[0,3] = [0,2) \cup [2,3]$

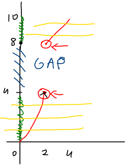
$F([0,3]) = F([0,2) \cup [2,3])$

$= F([0,2)) \cup F([2,3])$

$[0,4) \cup [8,9]$

? $= [F(0), F(3)] = [0,9]$

$F(0) = 0$, $F(3) = 9$



$F\big|_{[0,4]} : [0,4] \to [0,10]$

INJ, $\uparrow$, NOT INV

IF CODOMAIN IS $[0,4]$ OR $[9,?]$

$9 = F(3)$ IS OUTSIDE NOT COVD

WE RESTRICT THE CODOMAIN TO $[0,4) \cup [8,10]$

$G : [0,4] \longrightarrow [0,4) \cup [8,10]$ $\quad \uparrow$, $\underline{\underline{\text{INV}}}$

$$x \rightsquigarrow \begin{cases} x^2 & x \in [0,2) \quad 0 \leq x < 2 \\ x+6 & x \in [2,4] \quad 2 \leq x \leq 4 \end{cases}$$

$F(2) = 8$

WE HAVE $y = x^2$ IN $[0,2)$

$G\big|_{[0,2)} \equiv H : [0,2) \to [0,4)$

$x \rightsquigarrow x^2$

$G^{-1} : [0,2) \cup [8,10] \longrightarrow [0,4]$

$$x \rightsquigarrow \begin{cases} \sqrt{x} & x \in [0,2) \\ x-6 & x \in [8,10] \end{cases}$$

$H^{-1} : [0,4] \to [0,2]$

$x \rightsquigarrow \sqrt{x}$

$G\big|_{[2,4]} \equiv H : [2,4] \to [8,10]$

$x \rightsquigarrow x+6$

INVERSE COMPUTATIONS

$y = x+6$

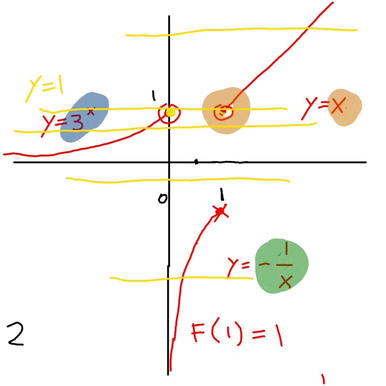
$x = y-6$

$G^{-1} : [8,10] \to [2,4]$

$x \rightsquigarrow x-6$

Ex $\quad F: \mathbb{R} \to \mathbb{R}$

$$x \rightsquigarrow \begin{cases} 3^x & x \le 0 \\ -1/x & 0 < x < 1 \\ x & x \ge 1 \end{cases}$$



I^{st}, $\uparrow$? NO $\quad F(0) = 1$

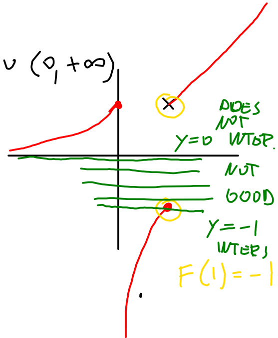
$\qquad\qquad F(1/2) = -2$

II^{nd}: INJ? NO Y=1 INTERSECTS TWICE $\qquad \to F(1/2) = -\frac{1}{1/2} = -2$

III^{rd}, IS SURJ? NO Y=2 DOES NOT INTERSECT!

$$F': \mathbb{R} \to \mathbb{R} - (-1, 0] = (-\infty, -1] \cup (0, +\infty)$$

$$x \rightsquigarrow \begin{cases} 3^x & x \le 0 \\ -1/x & 0 < x \le 1 \\ x & x > 1 \end{cases}$$



F' IS INJ $\qquad F'$ IS $\underline{\underline{\text{INV}}}$

NOT $\uparrow$

$$3x^2 - 2x - 2 > \sqrt{x} \qquad x \geq 0 \quad \text{E.F.}$$

$$\begin{cases} 3x^2 - 2x - 2 \geq 0 \\ x \geq 0 \\ 3x^2 - 2x - 2 > \sqrt{x} \end{cases} \cup \begin{cases} 3x^2 - 2x - 2 < 0 \\ x \geq 0 \\ 3x^2 - 2x - 2 > \sqrt{x} \end{cases}$$

NO SOLUTIONS

HERE WE CAN APPLY

$$F: \mathbb{R}_0^+ \to \mathbb{R}_0^+$$

$$x \rightsquigarrow x^2$$