FCS
Math: Functions
Exercises

Massimo Caboara

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Exercises, full solution will be provided

Exercise 1. *We have the function*

$$\begin{array}{rccc} F: & \mathbb{R} & \longrightarrow & \mathbb{R} \\ & x & \mapsto & 2x^2-3x-2 \end{array}$$

1. *Draw the graph of F. Mark the intersections with the axis and the vertex.*
2. *Determine $F([1,3])$.*
3. *Determine $F((-\infty,3])$.*
4. *Find $F^{-1}(-4)$, $F^{-1}(0)$, $F^{-1}(1)$, $F^{-1}(5)$.*
5. *Determine $F^{-1}([1,5])$.*
6. *Build an invertible function from F by restricting its domain and codomain.*
7. *Determine the formula for this inverse.*

Exercise 2. *We want to solve, with all the details, the inequality*

$$\log_3(x-2) < \log_3(3x+2)$$

with $x \in \mathbb{R}$

Exercise 3. *Given the sets*

$$A = \{n^2 \mid n \in \mathbb{N}\} \text{ and } B = \{k^3-1 \mid k \in \mathbb{N}\}$$

say if $\mid A \mid = \mid b \mid$ and find an explicit one-to-one correspondence.

Exercise 4. *Given the sets*

$A = \{(x,y) \in \mathbb{R}^2 \mid x = 1, y \in [1,2]\} \subset \mathbb{R}^2$ *and* $B = \{(x,y) \in \mathbb{R}^2 \mid x = \in [2,3], y = -1\} \subset \mathbb{R}^2$

say if $\mid A \mid = \mid b \mid$ and find an explicit one-to-one correspondence.

Exercise 5. *Given the sets*

$$A = \{(x, y) \in \mathbb{R}^2 \mid x = 1, y \in [1, 2]\} \subset \mathbb{R}^2 \text{ and } B = \{(k, 1) \mid k \in \mathbb{N}\} \subset \mathbb{R}^2$$

say if $| A | = | b |$ *and find an explicit one-to-one correspondence.*

Exercises

Exercise 6. *Are the following functions invertible? If they are, produce the inverse and draw its graph*

1. $$\begin{array}{rccc} f: & [-3/5, +\infty) & \longrightarrow & \mathbb{R}_0^+ \\ & x & \mapsto & 3\sqrt{5x+3} - 1 \end{array}$$

2. $$\begin{array}{rccc} f: & \mathbb{R} & \longrightarrow & (-pi/2, pi/2) \\ & x & \mapsto & \arctan -x + 2 \end{array}$$

3. $$\begin{array}{rccc} f: & [-3/5, +\infty) & \longrightarrow & \mathbb{R}_0^+ \\ & x & \mapsto & \sqrt{5x+3} \end{array}$$

4. $$\begin{array}{rccc} f: & \mathbb{R} & \longrightarrow & (-\pi/2, \pi/2) \\ & x & \mapsto & \frac{\arctan -x+2}{2} - 3 \end{array}$$

Exercise 7. *Determine which of the following functions are monotone by drawing the graphs.*

1. $$\begin{array}{rccc} f: & \mathbb{R} & \longrightarrow & \mathbb{R} \\ & x & \mapsto & \sin(x) \end{array}$$

2. $$\begin{array}{rccc} f: & \mathbb{R} & \longrightarrow & \mathbb{R} \\ & x & \mapsto & \tan(x) \end{array}$$

3. $$\begin{array}{rccc} f: & (-\pi/2, \pi/2) & \longrightarrow & \mathbb{R} \\ & x & \mapsto & \tan(x) \end{array}$$

4. $$\begin{array}{rccc} f: & (-\pi, \pi) & \longrightarrow & \mathbb{R} \\ & x & \mapsto & \sin(x) \end{array}$$

5. $$\begin{array}{rccc} f: & (0, \pi) & \longrightarrow & \mathbb{R} \\ & x & \mapsto & \cos(x) \end{array}$$

6.
$$\begin{array}{rccc} f: & \mathbb{R} & \longrightarrow & \mathbb{R} \\ & x & \mapsto & 5^x \end{array}$$

7.
$$\begin{array}{rccc} f: & \mathbb{R} & \longrightarrow & \mathbb{R} \\ & x & \mapsto & 1/5^x \end{array}$$

8.
$$\begin{array}{rccc} f: & \mathbb{R} & \longrightarrow & \mathbb{R} \\ & x & \mapsto & \arctan(x) \end{array}$$

9.
$$\begin{array}{rccc} f: & \mathbb{R}-\{0\} & \longrightarrow & \mathbb{R} \\ & x & \mapsto & 1/x^2 \end{array}$$

10.
$$\begin{array}{rccc} f: & \mathbb{R}-\{0\} & \longrightarrow & \mathbb{R} \\ & x & \mapsto & \log_{1/2}(x) \end{array}$$

Exercise 8. *Determine where the following functions are monotone.*

1.
$$\begin{array}{rccc} f: & \mathbb{R} & \longrightarrow & \mathbb{R} \\ & x & \mapsto & x^2+2x+1 \end{array}$$

2.
$$\begin{array}{rccc} f: & \mathbb{R} & \longrightarrow & \mathbb{R} \\ & x & \mapsto & -x^2+4x-3 \end{array}$$

3.
$$\begin{array}{rccc} f: & [0,4\pi] & \longrightarrow & \mathbb{R} \\ & x & \mapsto & -\sin(x) \end{array}$$

4.
$$\begin{array}{rccc} f: & [-\pi,\pi]-\{-\pi/2,\pi/2\} & \longrightarrow & \mathbb{R} \\ & x & \mapsto & \tan(x) \end{array}$$

Exercise 9. *Solve the following inequalities. If you use the function application method, remember to check that the function can be applied to the inequality, e.g. both parts of the inequalities belong to the function domain.*

1. $2^x < 2^{x^2+1}$
2. $\sqrt{x-3} > x$
3. $\frac{1}{x} < \frac{1}{x+2}$
4. $\log_3(x^2) < \log_3(x)$
5. $\left(\frac{1}{5}\right)^x > 5$
6. $\left(\frac{1}{5}\right)^{3x-1} > \left(\frac{1}{5}\right)^{2x+3}$
7. $\log_{1/2}(x-1) < \log_{1/2}(3x+1)$