FCS
Math: Functions
Exercises

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Exercises, full solution will be provided

Exercise 1. *Prove that* $| \mathcal{P}(\mathbb{N}) | = | \mathbb{R} |$ *[Solution will be discussed in class]*

Exercise 2. *We have the function*

$$F: \begin{array}{ccc} \mathbb{R} & \longrightarrow & \mathbb{R} \\ x & \mapsto & x^2 + 2x - 15 \end{array}$$

1. *Draw the graph of F. Mark the intersections with the axis and the vertex.*
2. *Determine $F([1, 2])$.*
3. *Determine $F((-\infty, 1])$.*
4. *Determine the intersection of the graph of F with the line $y = 0$.*
5. *Determine the intersection of the graph of F with the line $y = -7$.*
6. *Determine the intersection of the graph of F with the line $y = 1$.*
7. *Determine $F^{-1}([0, 1])$.*
8. *Determine $F([-5, +\infty))$.*
9. *Build an invertible function from F by restricting its domain and codomain.*
10. *Determine the formula for this inverse.*
11. *Find $F^{-1}(0)$, $F^{-1}(1)$, $F^{-1}(3)$, $F^{-1}(8)$.*

Exercise 3. *Is the function*

$$F: \begin{array}{ccc} \mathbb{R} & \longrightarrow & \mathbb{R} \\ x & \mapsto & e^{2x+1} - 3 \end{array}$$

injective, surjective, invertible? Can we make it invertibile by restriciting domain and/or codomain? In the latter case, what is the inverse function formula?

Exercise 4. *Is the function*

$$\begin{array}{rccc} T: & \mathbb{R} & \longrightarrow & \mathbb{R} \\ & x & \mapsto & \sqrt[3]{\sqrt[5]{x}+2} \end{array}$$

injective, surjective, invertible? Can we make it invertibile by restriciting domain and/or codomain? In the latter case, what is the inverse function formula?

Exercise 5. *We have the function*

$$\begin{array}{rccc} f: & \mathbb{R} & \longrightarrow & \mathbb{R} \\ & x & \mapsto & x^2 - 5x + 6 \end{array}$$

- *Determine* $f^{-1}([0, +\infty))$.
- *Determine* $f^{-1}(\mathbb{R})$.
- *Determine* $f^{-1}([-10, -20])$.
- *Determine* $f^{-1}([1, 3])$.

Exercise 6. *We have the function*

$$\begin{array}{rccc} F: & \mathbb{R} & \longrightarrow & \mathbb{R} \\ & x & \mapsto & \sin(x) \end{array}$$

- *Determine* $f^{-1}([0, 1])$.
- *Determine* $f^{-1}([0, \sqrt{2}/2])$.

Exercise 7. *We have the function*

$$\begin{array}{rccc} F: & \mathbb{R} & \longrightarrow & \mathbb{R}^+ \\ & x & \mapsto & 3^{x+2} \end{array}$$

- *Say why* F *is invertible.*
- *Find the explicit formula for* F^{-1}.
- *Determine* $F^{-1}([0, 1])$.
- *Determine* $F^{-1}([2, 4])$.
- *Determine* $F^{-1}([3, +\infty))$.
- *If* $a, b \in \mathbb{R}^+$, $a < b$, *determine* $F^{-1}([a, b])$

Example 1. *Consider the function*

$$\begin{array}{rccc} F: & \mathbb{Z} & \longrightarrow & \mathbb{Z} \\ & n & \mapsto & n^2 - n - 6 \end{array}$$

$$\begin{aligned} F^{-1}(\{0\}) &= ? \\ F^{-1}(\{1,2,3\}) &= ? \\ F^{-1}(\{-4,-6\}) &= ? \\ F^{-1}(\{0,1,2,3,4,5,6\}) &= ? \end{aligned}$$

Exercises

Exercise 8. *Is the function*

$$\begin{array}{rccc} f: & \mathbb{R} & \longrightarrow & \mathbb{R} \\ & x & \mapsto & \left(x^3 - 2\right)^5 \end{array}$$

injective, surjective, invertible? Can we make it invertibile by restriciting domain and/or codomain? In the latter case, what is the inverse function formula? [Invertible. Formula of the inverse is $\sqrt[3]{\sqrt[5]{x} - 2}$*]*

Exercise 9. *Is the function*

$$\begin{array}{rccc} T: & \mathbb{R} & \longrightarrow & \mathbb{R} \\ & x & \mapsto & 2x^2 + 3x - 2 \end{array}$$

injective, surjective, invertible? Can we make it invertibile by restriciting domain and/or codomain? In the latter case, what is the inverse function formula? [usual parabola exercise]

Exercise 10. *Is the function*

$$\begin{array}{rccc} f^{-1}: & [-3, +\infty) & \longrightarrow & \mathbb{R} \\ & x & \mapsto & \frac{\log_e(x+3) - 1}{2} \end{array}$$

injective, surjective, invertible? Can we make it invertibile by restriciting domain and/or codomain? In the latter case, what is the inverse function formula? [Invertible. Inverse formula is $3^{2x+1} - 3$*]*