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ON MAPS WHOSE DISTRIBUTIONAL JACOBIAN IS A MEASURE

Abstract

When the dimensions of domain and co-domain are the same, the Jacobian of a map is the determinant of its gradient. It was noticed long time ago that this nonlinear operator can be extended in a natural way to maps in certain Sobolev classes. This extension, known as distributional Jacobian, does not always agree with the Jacobian according to the standard (pointwise) definition, in the same way the distributional derivative of a function with bounded variation does not agree with the classical one. In particular, for map that take values in a given hypersurface, the pointwise Jacobian must vanish while the distributional one may not. If this is the case, the latter has an interesting interpretation in term of the (topological) singularity of the map. In this lecture I will review some of the basic results about maps whose distributional Jacobian is a measure, focusing in particular on maps with values in spheres.

The purpose of this lecture is to illustrate some recent work [2, 16, 17] on the structure of the distributional Jacobian for Sobolev maps $u : \mathbb{R}^n \to \mathbb{R}^k$, focusing in particular on its geometric interpretation when u takes values in the unit sphere. For a more detailed survey, see [1].

This research was mostly motivated by problems in the calculus of variations, and in particular the study of the asymptotic behaviour of certain functionals of Ginzburg-Landau type (see, e.g., [3, 6, 18, 20]). For lack of time, I will not discuss these applications here.

In the following sets and maps are always Borel measurable and "measure" means a (possibly vector-valued) σ-additive measure on Borel sets. When the measure is not mentioned, it is assumed to be the Lebesgue measure.

For the sake of exposition, it is convenient to present the results on the structure of the distributional Jacobian as generalizations of the well-known rectifiability theorem for the essential boundary of finite perimeter sets. There-

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fore I will first recall the definition of Sobolev and BV functions (§1 and §2), then describe the rectifiability theorem for finite perimeter sets (§3) and finally pass to the definition of Jacobian (§4 and §5) and to the corresponding rectifiability theorem (§7 and §8).

1. Sobolev functions [1]

Given a function $u : \mathbb{R}^n \to \mathbb{R}$, the *distributional derivative* of u belongs to L^p if there exist a map $g \in L^p(\mathbb{R}^n; \mathbb{R}^n)$ such that

$$\int_{\mathbb{R}^n} \mathrm{div}\varphi \, u = - \int_{\mathbb{R}^n} \varphi \cdot g \tag{1}$$

for every smooth test function $\varphi : \mathbb{R}^n \to \mathbb{R}^n$ with compact support, where divφ denotes the divergence of ϕ.

It is easy to check that the map g is essentially unique, and for smooth u it agrees (a.e.) with the usual gradient of u – indeed integration by parts implies that (1) is satisfied if one takes $g := Du$. Therefore g is called distributional derivative of u and denoted by Du. The Sobolev space $W^{1,p}(\mathbb{R}^n)$ consists of all functions $u \in L^p(\mathbb{R}^n)$ such that Du belongs to L^p; the Sobolev norm of u is $\|u\|_p + \|Du\|_p$.

Remark. – With an eye to applications to the calculus of variations and partial differential equations, one of the essential features of the Sobolev space $W^{1,p}(\mathbb{R}^n)$ is that it can be viewed as a closed subspace of $L^p(\mathbb{R}^n; \mathbb{R}^{n+1})$ via the natural embedding $u \mapsto (u, Du)$, and therefore it inherits all the compactness properties of the weak (or weak*) topology of L^p for $1 < p \leq \infty$. This paves the way to "easy" existence results for solutions of a large class of variational problems using the so-called direct method – that is, by semicontinuity and compactness arguments, cf. [7]. Note that $W^{1,1}$ does not have good compactness properties – its natural replacement is the space of BV defined in §2.

Since solution of variational problems and pde's are often very regular, the second natural step (typically the most difficult one) is to prove that the solutions obtained in certain Sobolev space are actually smooth. Here it is essential that Sobolev functions are not just elements of an abstract space obtained by completion – even though this is a possible way to define $W^{1,p}$ – but are really functions, and are differentiable almost everywhere (in the classical sense when $p > n$, and in a suitable approximate sense when $p \leq n$).

[1] For a detailed exposition of Sobolev functions see, e.g., [10, 21].

2. Functions with bounded variation [2]

Given $u : \mathbb{R}^n \to \mathbb{R}$, the distributional derivative Du is a measure if there exist an $\mathbb{R}^n$-valued measure μ on $\mathbb{R}^n$ such that (1) holds with g replaced by μ. The space of functions with bounded variations $BV(\mathbb{R}^n)$ consists of all $u \in L^1(\mathbb{R}^n)$ such that Du is a measure; the BV-norm of u is $\|u\|_1 + \|Du\|$, where $\|Du\|$ denotes the mass (or total variation) of the measure Du.

Remark. – For $n = 1$, the notion of function with bounded variation given in §2 is strictly related to the classical one. If $u : \mathbb{R} \to \mathbb{R}$ has bounded variation in the classical sense, then there exist a constant c and a real-valued measure μ on $\mathbb{R}$ such that $u(x) = c + \mu((-\infty, x])$ for every x where u is right-continuous. It follows immediately that $\int \varphi' u = - \int \varphi \, d\mu$ for every smooth test function φ with compact support, which means that u has bounded variation in the sense of distributions, and its distributional derivative is exactly the measure μ.

Conversely, if $u : \mathbb{R} \to \mathbb{R}$ has bounded variation in the sense of distributions, then it agrees almost everywhere with a function with bounded variation in the classical sense, and precisely the function $\tilde{u}(x) := c + Du((-\infty, x])$ where c is a suitably chosen constant and Du is the distributional derivative of u.

With this relation in mind, the following statement is a natural counterpart of the classical differentiability results for functions with bounded variation: a function $u \in BV(\mathbb{R}^n)$ is differentiable almost everywhere (in some approximate sense) and the gradient is the Radon-Nikodym density of the vector-valued measure Du with respect to the Lebesgue measure on $\mathbb{R}^n$.

3. Finite perimeter sets

A set $E \subset \mathbb{R}^n$ has *finite perimeter* if its characteristic function 1_E belongs to $BV(\mathbb{R}^n)$.

If the boundary of E is smooth, then the divergence theorem states that for every smooth $\varphi : \mathbb{R}^n \to \mathbb{R}^n$ with compact support there holds

$$\int_E \mathrm{div}\varphi = - \int_{\partial E} \varphi \cdot \nu \, d\mathscr{H}^{n-1} \ , \tag{2}$$

where ν is the *inner normal* to ∂E and $\mathscr{H}^{n-1}$ denotes the $(n-1)$-dimensional Hausdorff measure. Comparing (1) and (2) we deduce that the distributional derivative of 1_E is given by the restriction of the measure $\mathscr{H}^{n-1}$ to the boundary of E, multiplied by the vector-valued density ν, that is,

$$D1_E = \nu \, 1_{\partial E} \, \mathscr{H}^{n-1} \ . \tag{3}$$

[2] For a detailed exposition see [4, 10, 14].

If E is an arbitrary set with finite perimeter, then formula (3) still holds provided that ∂E is replaced by the *essential boundary* $\partial_* E$ – namely the set of all points of $\mathbb{R}^n$ where the density of E is $1/2$ – and the inner normal ν is defined in a suitable approximate sense. Moreover the essential boundary $\partial_* E$ is $(n-1)$-rectifiable, that is, it can be covered, except for an $\mathscr{H}^{n-1}$-negligible subset, by countably many hypersurfaces of class C^1. This result is known as the structure theorem for finite perimeter sets; in this form it is due to E. De Giorgi [8, 9] and H. Federer [11].

Remark. – The structure theorem shows that the notion of finite perimeter set is a natural generalization of that of smooth set; moreover the class of finite perimeter sets inherits from BV good compactness properties, and therefore it was largely used to prove existence results for minimal hypersurfaces (cf. [14]).

4. Jacobian of smooth maps

Given $k \le n$ and a map $u : \mathbb{R}^n \to \mathbb{R}^k$ of class C^1, we call *Jacobian* of u the pull-back of the volume form on $\mathbb{R}^k$ according to u, that is, the k-form on $\mathbb{R}^n$ given by

$$Ju := du_1 \wedge \ldots \wedge du_k \ , \tag{4}$$

where u_i denotes the i-th component of u and the 1-form du_i is defined by

$$du_i := \frac{\partial u_i}{\partial x_1}\, dx_1 + \cdots + \frac{\partial u_i}{\partial x_n}\, dx_n \ .$$

If we identify n-forms on $\mathbb{R}^n$ with functions, for $k = n$ formula (4) becomes

$$Ju := \det(Du) \ . \tag{5}$$

Note that formula (4) makes sense even for maps of class $W^{1,p}$ with $p \ge k$; in this case the Jacobian is a continuous nonlinear operator from $W^{1,p}$ to the space of k-forms on $\mathbb{R}^n$ with coefficients in $L^{p/k}$ – each du_i belongs to L^p and therefore the product $du_1 \wedge \ldots \wedge du_k$ belongs to $L^{p/k}$.[3]

5. Distributional Jacobian

If u is an arbitrary map in $W^{1,p}$ with $p < k$ the right-hand side of (4) may not make any sense, not even as a distribution. However, it is possible to write the Jacobian in different ways, and some of these are well-posed even for maps u in $W^{1,p}$ for some $p < k$.

[3] The continuity justifies the claim that this is the "right" extension of the Jacobian operator to these Sobolev classes.

For instance, for smooth maps $u : \mathbb{R}^2 \to \mathbb{R}^2$ a simple computation yields

$$Ju = \det(Du) = \frac{\partial}{\partial x_1}\left(u_1 \frac{\partial u_2}{\partial x_2}\right) - \frac{\partial}{\partial x_2}\left(u_1 \frac{\partial u_2}{\partial x_1}\right) \ . \tag{6}$$

Note that the products between brackets are well-defined if $u_1 \in L^\infty$ and $Du_2 \in L^1$ (a condition essentially weaker than $Du \in L^2$) and therefore their partial derivatives makes sense at least as distributions. Hence formula (6) can be used to define the Jacobian for maps $u : \mathbb{R}^2 \to \mathbb{R}^2$ of class $L^\infty \cap W^{1,p}$ with $p \ge 1$.[4]

A general (and more symmetric!) formula is the following: for smooth maps $u : \mathbb{R}^n \to \mathbb{R}^k$ there holds

$$Ju = \frac{1}{k} d\Bigg(\sum_{i=1}^{k} (-1)^{i-1} u_i \bigwedge_{j \ne i} du_j\Bigg) \ . \tag{7}$$

Note that when $u \in L^\infty$ and $Du \in L^{k-1}$ the form between brackets is a well-defined $(k-1)$-form with coefficients in L^1, and its differential is a well-defined k-form with coefficients in the space of distributions.

Following the work of R. Jerrard and H.M. Soner [16, 17], we use (7) as definition of the *distributional Jacobian* for maps $u : \mathbb{R}^n \to \mathbb{R}^k$ of class $L^\infty \cap W^{1,p}$ with $p \ge k-1$. Thus the distributional Jacobian is a continuous nonlinear operator from $L^\infty \cap W^{1,p}$ with $p \ge k-1$ into the space of k-forms whose coefficients are distributions. Continuity and the density of smooth functions ensure that the distributional Jacobian agrees with the pointwise Jacobian defined in (4) for all maps in $L^\infty \cap W^{1,p}$ with $p \ge k$.

Remark. – The distributional Jacobian can be defined also for maps u on some open set Ω in $\mathbb{R}^n$ and which are *locally* of class $L^\infty \cap W^{1,p}$ with $p \ge k-1$. On the other hand, when $p < k-1$, examples show that there is no extension of the Jacobian operator to any reasonable class of maps with derivative in L^p which is continuous with respect to any reasonable choice of the topologies.

6. A fundamental example

To understand the relevance of the distributional definition, consider the following example: Ω is the unit ball in $\mathbb{R}^2$ centered at the origin, and $u : \Omega \to \mathbb{R}^2$ is the map given by

$$u(x) := \frac{x}{|x|} \ .$$

[4] This is a particular case of the definition of distributional determinant introduced in [5].

Then u is smooth away from the origin, and the distributional derivative Du agrees with the classical one and belongs to L^p for every $p < 2$, but not to L^2. An explicit computation shows that the determinant of Du is always null.

On the other hand, the distributional Jacobian is π times the Dirac mass centered at the origin: this can be easily verified by computing the limit of the Jacobians for a sequence of regular maps u_ε that converge to u in $W^{1,1}$, for instance

$$u_\varepsilon(x) := \begin{cases} x/\varepsilon & \text{if } |x| \le \varepsilon \\ x/|x| & \text{if } |x| > \varepsilon \end{cases} .$$

Hence the distributional Jacobian defined in (6), or equivalently in (7), does not agree with the pointwise definition in (5), even though the latter makes perfect sense for this particular map.

Remark. – A somewhat similar phenomenon occurs for functions of one variable with bounded variation: if $u(x) := \mathrm{sgn}(x)$ then the derivative u' exists and is null at every point except the origin, while the distributional derivative of u is 2 times the Dirac mass at the origin.

7. Jacobian of maps with values in spheres I

We restrict now our attention to maps $u \in W^{1,p}(\mathbb{R}^n;\mathbb{R}^k)$ that satisfy $|u| = 1$ everywhere, that is, maps with values in the sphere S^{k-1}. We will see that when the distributional Jacobian of these maps is a measure, then its structure is quite similar to the structure of the distributional derivative of (characteristic functions of) finite perimeters described in §3.[5]

We consider first the case $k = n$. Recall that for maps from $\mathbb{R}^n$ to $\mathbb{R}^n$ the Jacobian is an n-form on $\mathbb{R}^n$ (possibly with coefficients in the space of distributions) that we tacitly identify with a function (a distribution).

Since u takes values in an $(n-1)$-dimensional submanifold of $\mathbb{R}^n$, the determinant of Du must be null at every point where u is (approximately) differentiable. If $p \ge n$, this implies $Ju = 0$. On the other hand, the example in §6 shows that this may be no longer true when $n - 1 \le p < n$ and Ju is the distributional Jacobian. That example is actually a particular case of a more general result (cf. [6, 17]): if u is smooth outside a finite singular set $E = \{x_i\}$, then

$$Ju = \alpha_n \sum_i d_i \, \delta_{x_i} \,, \tag{8}$$

where α_n is the volume of the unit ball in $\mathbb{R}^n$, δ_{x_i} is the Dirac mass centered at the point x_i, and d_i is the *Brower degree* of the restriction of u to an arbitrary $(n-1)$-dimensional sphere S_i that does not intersect the set E and encloses x_i but no other point of E. In particular d_i is an integer.

What is more interesting, the validity of formula (8) is not limited to the particular class of maps considered above: for *every* map $u : \mathbb{R}^n \to S^{n-1}$ of class $W^{1,p}$ with $p \ge n-1$, if the distributional Jacobian Ju is a measure then it can be represented as in (8) for a suitable choice of finitely many points x_i and integers d_i.[6] The point x_i can still be interpreted (in a sense that will not be made precise here) as singularities of u, and the numbers d_i as degrees of the restriction of u to suitable spheres. However, these x_i are not all the points of discontinuity or even approximate discontinuity of u (which can be infinite and even dense in $\mathbb{R}^n$) but correspond to singularities that are topologically necessary; in particular they are stable under small perturbations in the class of maps from $\mathbb{R}^n$ to S^{n-1} ("small" is intended in the sense of the Sobolev norm).

8. Jacobian of maps with values in spheres II

The results in §7 can be extended to maps with values in S^{k-1} for every $k < n$.

In the general case, however, precise statements require a certain amount of notation from multilinear algebra, and can be properly understood only in the framework of the theory of currents (see [13, 12, 19]). For this reason we begin with the case $k = n-1$, that is, maps $u \in W^{1,p}(\mathbb{R}^n;\mathbb{R}^{n-1})$ such that $|u| = 1$ everywhere.

For maps from $\mathbb{R}^n$ to $\mathbb{R}^{n-1}$ the Jacobian is an $(n-1)$-form on $\mathbb{R}^n$, that we tacitly identify with a vector-field on $\mathbb{R}^n$.

As before, if u takes values in S^{n-2} the rank of Du must be strictly less than $n-1$ at every point where u is differentiable, and therefore the pointwise Jacobian in (4) vanishes. However, for $n-2 \le p < n-1$ the distributional Jacobian may not vanish: if u is smooth outside a singular set E given by a finite union of smooth curves M_i which are pairwise disjoint, closed, and oriented, then Ju is the $\mathbb{R}^n$-valued measure given by

$$Ju := \alpha_{n-1} \sum_i d_i \, \tau_i \, 1_{M_i} \, \mathscr{H}^1 \,, \tag{9}$$

where $1_{M_i} \mathscr{H}^1$ is the restriction of the 1-dimensional Hausdorff measure to the curve M_i, τ_i is the unit tangent vector-field orienting M_i, and d_i is the Brower degree of the restriction of u to any $(n-2)$-dimensional sphere S_i in $\mathbb{R}^n$ that does not intersect the singular set E, and whose *winding number* around M_j is 1 for $j = i$ and 0 for $j \ne i$.

[5] Which is not entirely surprising, since characteristic functions can be viewed as maps with values in the zero-dimensional sphere.

[6] This is a particular case of a general result stated in §8.

As in the case $k = n$, it can be shown that given *any* map $u : \mathbb{R}^n \to S^{n-2}$ of class $W^{1,p}$ with $p \geq n-2$, if the distributional Jacobian Ju is a measure, then it can be represented as in (9) for a suitable choice of countably many rectifiable closed curves M_i (not necessarily pairwise disjoint) and integers d_i. As far as I know, at the moment there is no precise interpretation of the curves M_i in terms of singularities of the map u.

Finally, we briefly describe what happens for general k: if $u : \mathbb{R}^n \to S^{k-1}$ is a map of class $W^{1,p}$ with $p \geq k-1$ which is smooth outside a finite union of $(n-k)$-dimensional surfaces M_i that are connected, oriented, and pairwise disjoint, then the following generalization of (9) holds (cf. [2, 17]):

$$Ju := \alpha_k \sum_i d_i \, \tau_i \, 1_{M_i} \, \mathscr{H}^{n-k} , \tag{10}$$

where the k-form Ju is now identified with an $(n-k)$-dimensional current (a distribution with values in $(n-k)$-multivectors), τ_i is the $(n-k)$-multivector orienting M_i, and the numbers d_i are the degrees of the restriction of u to suitable $(k-1)$-dimensional spheres S_i.

For an arbitrary map $u : \mathbb{R}^n \to S^{k-1}$ of class $W^{1,p}$, $p \geq k-1$, the following structure theorem holds (see [2, 15, 16]): if the distributional Jacobian Ju is a measure, then

$$Ju := \alpha_k \, d \, \tau \, 1_M \, \mathscr{H}^{n-k} , \tag{11}$$

where M is an $(n-k)$-dimensional rectifiable set, d is an integer-valued multiplicity function, and τ is an $(n-k)$-multivector spanning the approximate tangent space of M.

More precisely, it was proved in [2] that Ju agrees, up to the factor α_k, with the *boundary* of a generic level set of u (which can be naturally viewed as an $(n-k+1)$-dimensional *integral current*). Therefore the boundary rectifiability theorem by Federer and Fleming yields that Ju agrees, up to the factor α_k, with an $(n-k)$-dimensional integral current without boundary.

9. Further results

Concerning maps with values in spheres, it is natural to ask whether the results described in the previous paragraph characterize the image of the Jacobian operator. This problem has been studied extensively in [2].

We can formulate the question as follows: given an integer d and a compact, connected, oriented surface M with dimension $n-k$ in $\mathbb{R}^n$ and without boundary, is it possible to find a map $u : \mathbb{R}^n \to S^{k-1}$ of class $W^{1,k-1}$, smooth in the complement of M, such that formula (10) holds, that is, the degree of the restriction of u to S is equal to d for any $(k-1)$-dimensional sphere S contained in $\mathbb{R}^n \setminus M$ whose winding number around M is 1?

It was proved in [2] that the answer is positive in general only for $k = 2$ (note that this result is not immediate even when $n = 3$ and M is the usual threefold knot). It was also shown that for $k > 2$ the answer becomes positive if one allows the required map u to be smooth only in the complement of $M \cup E$, where E is an additional singularity with dimension $n-k-1$.

In [2] a precise characterization of the image of the Jacobian operator is also given: given an $(n-k)$-dimensional current T in $\mathbb{R}^d$, then there exists a map $u : \mathbb{R}^n \to S^{k-1}$ of class $W^{1,k-1}$ such that the distributional Jacobian of u satisfies $Ju = \alpha_k T$ (cf. formula (11)) if and only if T is the boundary of a rectifiable current with integral multiplicity and finite mass in $\mathbb{R}^n$.

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