Logical independencies in modal logic

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A propositional modal logic is formulated in which a wider range of relations of logical dependence and independence can be syntactically represented than in basic modal logic (**ML**). This logic, originally introduced in [4], is referred to as *Independence-friendly (IF) modal logic* (or **IFML**): It was designed to be a 'modal logical analogue' of the Independence-friendly first-order logic of Hintikka and Sandu (cf. e.g. [1, 2, 6]).

The semantics of **IFML** is formulated making use of two-player evaluation games; the players of these games may be called *Verifier* and *Falsifier*. The former is responsible for interpreting the operators $\Diamond$ and $\vee$, while the latter is to interpret the operators $\Box$ and $\wedge$. The existence of a winning strategy for *Verifier* (resp. *Falsifier*) in a game $G(\varphi, \mathcal{M}, w)$ associated with a formula φ and a pointed modal structure $(\mathcal{M}, w)$ is equated with the truth (*resp.* falsity) of φ in $\mathcal{M}$ at w.

Logical independence of, say, a diamond $\Diamond$ from a set $\{O_i : i \in W\}$ of syntactically superordinate operators is modeled by the requirement that a strategy f of *Verifier* – to be winning – must be *uniform* with respect to the interpretations of these operators O_i $(i \in W)$: if h and h' are any two plays of the relevant semantical game at which *Verifier* is to interpret O, differing at most for the interpretations of the operators operators O_i $(i \in W)$, then $f(h) = f(h')$. It turns out that unlike for basic modal logic, there are evaluation games of IF modal logic in which *neither* of the players has a winning strategy. Such games are termed *indeterminate*.

A survey is given on IF modal logic and its known properties. Emphasis is put on showing how extra logical independencies are introduced to the

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language of basic modal logic. It is pointed out that according to the choice of syntax (i.e. the decision which operators are allowed to be independent from which others) big differences in the expressiveness of the resulting logics arise. Finally, known metalogical results on **IFML** are mentioned (cf. [3, 4, 5]), including the facts that **IFML** is strictly more expressive than **ML** relative to arbitrary modal structures; that **IFML** has a translation into first-order logic but not to any of its finite-variable fragments; that *Verifier*'s winning strategies in evaluation games of **IFML** admit of a history-free normal form; and that the satisfiability and validity problems of **IFML** are not each other's duals but are both decidable.

References

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