VAUGHT CONJECTURE AND MODULES WITH FEW TYPES

Vera Puninskaya

Dipartimento di Matematica e Fisica – Università di Camerino

vera.puninskaya@unicam.it

The following statement is known as Vaught's conjecture:

Conjecture 0.1 (Vaught's conjecture). *Let T be a countable complete first–order theory. If T has uncountably many non–isomorphic countable models, then T has 2^ω non–isomorphic countable models.*

It was proved for ω-stable theories by Shelah in 1984. Garavaglia proved Vaught's conjecture for ω-stable theories of modules in 1980.

The main objective of this talk is to show that Vaught's conjecture is true for any complete theory of modules over a countable serial ring, over a countable commutative Prüfer ring, over a countable hereditary noetherian prime ring and over a tame hereditary finite–dimensional algebra over a countable infinite field. In all these cases, it was shown that a theory with few models is ω-stable thus making it possible to apply Garavaglia's result. Also in the talk will be obtained a description of modules with few models over a countable serial ring and a description of modules with few types over a countable commutative Prüfer ring. It will be shown thta Vaught's conjecture is true for modules over a path algebra $K\Lambda_2$ over a finite or countable field K. It will be received a complete description of modules with few types and of Σ-pure–injective modules over the path algebra $K\Lambda_2$ and over a tame hereditary finite–dimensional algebra over a countable or finite field K. For instance, it follows that Vaught's conjecture is true for any complete theory of modules over a tame hereditary finite–dimensional algebra over a finite field K.

There are also some results on Vaught's conjecture for modules over a group ring and for modules over a pullback ring.