

Characterizing models by means of their internal language

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Abstract

The concept of internal language of a structure is widely used in category theory to reason logically within a categorical structure.

The categorical structure of a topos constitutes a main example for which this happens: indeed each topos can be viewed as a generalized universe of sets thanks to its internal language formulated in terms of a many sorted logic (see [LS86, MM92]) or a dependent type theory (see [Mai01]). Moreover, we know internal languages of many other categorical structures (see [Mai01]).

The internal language concept is useful to establish a link between a logical calculus and a class of structures stronger than soundness and completeness theorems. Indeed, while we can have soundness and completeness theorems of a logic with respect to different not equivalent structures, a logical calculus with its theories provides the internal language of a class of structures up to suitable equivalences.

We can apply the internal language concept to classify different classes of complete models for intuitionistic linear logic and relate them via categorical adjunctions (see [MMPR]).

A further step would be to perform the same analysis for models of intuitionistic (or classical) predicative many sorted logics.

References

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