ω-categorical weakly o-minimal expansions of Boolean lattices

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Let $\mathcal{A} = (A, \leq, \ldots)$ expand a Boolean lattice $(A, \leq)$. $\mathcal{A}$ is called *weakly o-minimal* if and only if each definable subset of A definable in $\mathcal{A}$ is a Boolean combination of convex sublattices. So weak o-minimality naturally extends in this framework the well known notion for expansions of total orderings (see [MMS]).
Examples include

- atomless Boolean lattices $(A, \leq)$ (they are o-minimal, too);
- any expansion of an atomless Boolean lattice by a maximal ideal I (this is not o-minimal when I is not principal).

We are interested in studying ω-categoricity and related notions in this framework (compare with [HMMNT] in the linear case).

Theorem 1. *Let $\mathcal{I}$ be a finite sequence of ideals of a Boolean lattice $(A, \leq)$ closed under the Heyting algebra operations. Then the following are equivalent for $\mathcal{A} = (A, \leq, \mathcal{I})$:*

(*i*) *$\mathcal{A}$ is weakly o-minimal;*

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(*ii*) $\mathcal{A}$ *is* ω*-categorical;*

(*iii*) $\mathcal{A}/I$ *has only finitely many atoms for every* $I \in \mathcal{I}$.

Recall that a complete countable theory T is called *p-ω-categorical* if and only if the Boolean algebras of definable sets of countable models of T are pairwise isomorphic.

Theorem 2. *Any complete theory of weakly o-minimal expansions of Boolean lattices is p-ω-categorical.*

References

[LT1] S. LEONESI, C. TOFFALORI, Omega-categorical Weakly o-minimal expansions of Boolean Lattices, *Mathematical Logic Quarterly*, 49 (2003) n.4.

[LT2] S. LEONESI, C. TOFFALORI, On the Boolean Algebras of Definable Sets in Weakly o-minimal Theories, *Math. Logic Quarterly* 50 (2004) n.3.

[HMMNT] B. Herwig, H.D. Macpherson, G. Martin, A. Nurtazin, J.K. Truss, "On $\aleph_0$-categorical weakly o-minimal structures", *Annals of Pure and Applied Logic* 101 (2000) n.1.

[MMS] D. Macpherson, D. Marker, C. Steinhorn, "Weakly o-minimal structures and real closed fields", *Trans. Amer. Math. Soc.* 352 (2000) n.12.