KOLMOGOROFF AND THE RELEVANCE OF THE DOUBLE NEGATION LAW IN SCIENCE

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By Brouwer intuitionistic thinking on logic was presented as a distrust on the classical theory of logic which had to be quickly dethroned. Even in 1927, he wrote a paper asking formalists to concede four points, three out them constituted a very surrender by formalists. In order to support his rejection of classical logic, Brouwer's arguments relied upon the possible failure of the LEM. He presented a merely starting point for a new kind of logic.

Instead, in 1925, K.'s paper on LEM wanted to explore both kinds of logic - the classical one CL and the intuitionistic one -, in particular, which point characterises their logical difference. K.'s emphasises the key-role played by the DNL rather than by the LEM in marking the borderline between the two kinds of logic. Moreover, IL was presented as a new system of logic, merely drawn form a different viewpoint than the classical logic's one, yet insisting on the same reality. In other terms, after Kolmogoroff IL changed in nature; no more a matter of metaphysics, but a matter of choice on the scientific viewpoint. As a consequence, its difference with CL can be expressed at least through the failures of some logical laws. This new perspective, i.e. a conflictual co-existence of two kinds of logic, was shared by intuitionists not before ten years after. Moreover, the role played by DNL rather than LEM as the very borderline between the two kinds of logic, was eventually recognised by Dummett and Prawitz in 70's.

The change from LEM to DNL in focussing the relationship between the two kinds of logic have implications which cannot be underemphasized. To recognize a failure of LEM in a scientific text constitutes even a sophisticated work; instead, it is easy to recognize an occurrence of a double negated statement; one easily remark a statement including a negation; then one has to verify whether this sentence is not equivalent to the corresponding, positive one, owing to the lack of scientific evidence of the latter one (DNS).

Since ten years, I scrutinized several original texts founding theories belonging to Mathematics, Physics and Chemistry in order to notice all the occurrences of DNS's. In addition, one can extract from an original text all DNS's and list them in the presentation order. In the cases of Lobachevsky's non-Euclidean geometry, S. Carnot's thermodynamics and Avogadro's atomism, each of these lists is enough for representing the whole logical development of the corresponding theory. Moreover, each list of DNS's can be severed in some units of arguing; each one of them can be considered as a cycle of arguing when it ends by means of an *ad absurdum* argument. This kind of argument is rejected by IL, since in the common use of it, the last step of the argument applies the classical double negation law. But one can refrain from performing such a step and to consider the argument as concluded by the last DNS, provided that the theoretical context is different from an axiomatic one; in this case the last DNS constitutes a methodological principle for subsequently developing a new cycle of argument. As a whole the theory is aimed to discover a scientific method, which can solve the universal problem motivating the theory at issue (say, how much parallel lines exist).

Even K.'s paper can be scrutinized under this light. Although it argues upon two axiomatic systems, surely its context of arguing is not an axiomatic one. Rather, K. declares that he wants to

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solve an universal problem in logic, whether transfinite arguments lead to contradictions. Then, he invents a new scientific method for solving it by scrutinizing the relationship between the two logical systems. In fact, K.'s paper includes DNS's. They are mainly located in the introductory part, just where K. is presenting the problem (in the fourth period of the paper). In this case too the list of DNS's is enough for representing the logical core of the paper. In particular an *ad absurdum* theorem is verbally given in sect. V.B in order to disprove a Brouwer's thesis; never the use of LEM leads to a contradiction

The relevance of DNL for the foundations of logic in general is supported by one more result concerning an extreme kind of formalization of logic, i.e. paraconsistent logic. In a previous paper I showed that Vasiliev's paraconsistent logic can be interpreted according to a DN translation of the three characteristic Vasiliev's sentences "S is A", "S is not A", "S is and is not A". One can to substitute the word "is" by means of the implication symbol $\rightarrow$, and A by $\neg\neg$S. Then, one obtains the three statements representing the three characteristic roles played inside a theoretical context by a statement - which is insufficiently supported by empirical evidence -, when considered as either a principle; i.e., either an axiom-principle, or a methodological heuristic principle, or a guess whose role is not provisionally decided.

K.'s new attitude gave a linkage between logic and foundations of science. This linkage is manifested by two facts. First, 1925 K.'s paper was able to obtain an important result outside the field of logic, i.e. the anticipation of the translatability of classical mathematics to intuitionistic mathematics. Then, 1932 K.'s paper, by abandoning the viewpoint of a subjectivist metaphysical basis, offered a new interpretation of IL by referring it to an objective basis, i.e. a community activity for solving problems, e.g. a classroom activity. This interpretation takes its full meaning when its basic notions are referred to a more wide context; i.e. when a problem is considered as a scientific problem of an universal nature and at the same time the community activity is seen as the scientific research performed by the scientists community, in particular the logicians. Under this light, IL manifests itself as the specific logic governing the heuristic organization of a scientific theory, i.e. the kind of organization which had been called 'empirical' by both d'Alembert and L. Carnot, and which one century after was called 'a principle theory' by both Poincaré and Einstein.

One has to conclude that K., by emphasizing the theoretical role played by DNS's, addressed logic to enlighten rather than one feature only of science foundations - the axiomatic organisation whose deductive method is governed by CL -, directly science foundations in the widest sense. Moreover, K.'s paper first introduced the logical research into the foundations of science. He discovered a very structure of the foundations of the whole science; indeed the failure of DNL leads to consider the global context of a scientific theory, i.e. the whole organisation of a theory. A very fruitful change in the foundational attitude resulted in several fields of scientific research. I substantiate this conclusion by adding the philosophical consequences of 1925 paper for the foundations of science.