Topological extensions, ultrafilters and nonstandard analysis

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[Joint work with Marco Forti].

We introduce the notion of *topological extension* $^*X$ of a given set X. It can be considered as a sort of "completion" or "compactification" where the existence of a continuous extension $^*f : {}^*X \to {}^*X$ is postulated for each function $f : X \to X$. The resulting topological spaces include the Stone-Čech compactification βX of the discrete space X, as well as all nonstandard models of X in the sense of nonstandard analysis (when endowed with a "natural" topology). In particular, we obtain a simple characterization of nonstandard models in purely topological terms.

The existence of Hausdorff (T_2) topological extensions is equivalent to the existence of a special class of ultrafilters, named *Hausdorff ultrafilters*, which generalize *selective* ultrafilters. Thus, the existence of Hausdorff topological extension follows from the *continuum hypothesis* (or even by *Martin's axiom*). The independence of Hausdorff ultrafilters from the axioms of ZFC has been recently proved by Bartosynski and Shelah.